# Specificity of Undivided Time Series Forecasting Described with Innovation Curves

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Abstract: In this paper, we consider characteristics of remote time period forecasting in the case when particular time periods could be described with innovation curves. Also, time series, whose periods determined by the curve types known, are not clearly seen. However, the fact that the time series describe projects with the same product evolution is known, and the time when new generations appear is determined.

## 1 INTRODUCTION

It is well known that many production and economic systems parameters are described with special curves. The type of curve, which describes the parameter, depends on that parameter. Innovation curves describe economic indexes, e.g. profits, sales volume, market share, which is held by company or product, a number of rival firms or products, how many people are recruited in the project work, product quality, etc. Engineering and process-dependent parameters are described with S-Curves, e.g. developing and the introduction of the new technology, performance index, the degree of maturity of the technology or process, etc. The S-Curves show the degree of the technology development and prospects of its modernization. Each parameter of the innovation project could be at its developmental stage and described its own functional dependence.

Issues, concerned with conceptual modeling (Mylnikov, 2015) and prognostic model development for management tasks solving in production and economic systems to remote time periods, are becoming more and more relevant in view of business conducting under market conditions. There is no way to develop these models without employing parameters predictions. And in this case, when considering the long-term statistics, as a rule, attention is not paid to revision and product modification peculiarities as it is considered as insignificant. However in terms of market modifications are individual products with its own life cycle properties concerned with modernization level, different from the main products one and which are in functional dependence on main parameters.

In order to forecast undivided time series, approaches based on the fitting criteria and fractal methods of forecasting are traditionally used. The fractal approach requires a lot of statistics. This characteristic makes it impossible to analyze financial reporting (Crownover, 1995). Moreover, the fractal methods are best suited to describe the parameters, which are characterized by chaotic changing (Feder, 1988). Modeling based on fitting criteria has the low precision of describing in the case when the product has several generations (Bjorck, 1996). For mathematical formulation, in this case, could be applied dynamic time warping.

## 2 DATA PREPARATION FOR SOLVING FORECAST TASKS

To test the hypothesis we construct the model using Ford concern's historical data of various car generations and models for various periods. For that, we take sales volume and pricing data from official concern [corporate.ford.com/investors/reports-and-filings/monthly-reports.html] (monthly reports from 2012 to 2016) and supplement them with data from computational knowledge engine WolframAlpha [www.wolframalpha.com].

Table 1: The statistics of Ford Expedition price and sales volume changing.

| Year | Expedition | | | |
|---|---|---|---|---|
| | Sales volume | Price $ (min) | Price $ (med) | Price $ (max) |
| 1997 | 45974 | 27620 | 30914 | 34225 |
| 1998 | 214524 | 28225 | 31449 | 34690 |
| 1999 | 225703 | 29355 | 33989 | 39095 |
| 2000 | 233125 | 29845 | 34645 | 39885 |
| 2001 | 213483 | 30195 | 35204 | 40850 |
| 2002 | 178045 | 30555 | 35501 | 41085 |
| 2003 | 163454 | 31820 | 36763 | 41560 |
| 2004 | 181547 | 32500 | 36870 | 41995 |
| 2005 | 159846 | 33455 | 39703 | 46340 |
| 2006 | 114137 | 32660 | 38815 | 45240 |
| 2007 | 87203 | 29245 | 35445 | 39995 |
| 2008 | 90287 | 31345 | 38294 | 43590 |
| 2009 | 55123 | 34845 | 42196 | 47850 |
| 2010 | 31655 | 35585 | 42906 | 48590 |
| 2011 | 37336 | 36205 | 43265 | 49655 |
| 2012 | 40499 | 36530 | 43380 | 49680 |
| 2013 | 38062 | 40605 | 46763 | 51355 |
| 2014 | 38350 | 41975 | 49558 | 56205 |
| 2015 | 44632 | 43845 | 55257 | 62410 |
| 2016 | 41443 | 45435 | 56563 | 63375 |

Production and economic system's parameters could be defined with output goods' parameters. Two parameters: sales volume and the current price could be used for simplified estimation of competitiveness. Sales volume reflects customers' preferences and could be a criterion of competitiveness. The product price allows estimating proceeds and profit of the system. The ratio of profit and sales volume also characterize business and economic activity efficiency of the system. It is possible to make initial forecast of the system's parameter behavior with the help of received data. Production and economic system's parameters describing might be more accurate when more parameters adding. Consequently, their forecasting also improves.

For conducting research we collected data about models Ford Expedition, C-Max (01.01.2013 – 01.10.2016), Edge (01.01.2007 – 01.10.2016), Escape (01.01.2001 – 01.10.2016), Expedition (01.01.1997 – 01.10.2016), Explorer (01.01.1991 – 01.10.2016), Fiesta (01.01.2013 – 01.10.2016), Focus (01.01.2000 – 01.10.2016), Fusion (01.01.2006 – 01.10.2016), Mustang (01.01.1991 – 01.10.2016), Transit (01.01.2015 – 01.10.2016). To work with the model we represent Ford Expedition data in table 1.

Distinctive feature of the data is that there are several model generations in considered time periods. Besides, we know periods of simultaneous production of old and new model generations. Sales volume and price changing of each model could be described with the innovation curve (Mylnikov, 2013). However, the whole time series cannot be described with the curve. Therefore we make innovation and S-Curve for each generation each model.

## 3 PIECEWISE FUNCTION DESCRIPTION OF CONTINUOUS TIME SERIES USING INNOVATION CURVE

Innovation curve is a piecewise function and consists of exponential growth, linear growth, and parabolic maternity piece. Set of equations describing the curve:

$$\begin{cases} f_1(t) = e^{c_0 t}, & 0 < t < t_1 \\ f_2(t) = c_1 + c_2 t, & t_1 < t < t_2 \\ f_3(t) = c_3 + c_4 t + c_5 t^2, & t_2 < t < t_3 \end{cases}$$

Factors $c$ define the function's position, shape and increasing (decreasing) and depend on innovation project specifics. Values $t_1, t_2, t_3$ correspond to time values of transition points between innovation project stages.

The problem of determining the transition points could be solved with expert method based on innovation curve characteristics, such as (Wolberg, 2006): the area bounding with exponential growth piece (market entry piece) is 3% of the total figure area; the area bounding with linear growth piece is 13% of the total figure area; the area bounding with parabolic maternity piece is 34% of the total figure area; the area bounding with decline piece is 16% of the total figure area.

When calculating the data collected, we neglect the first growth stages of some models because of short entry market period or its absence and also the inadequate amount of date during the life cycle of the models under consideration.

To fulfill the conditions of the curve smoothness it is necessary to provide coincidence of value of functions and their first-order derivative. For that, we could use known function transition point's values. As the result of initial data analysis we could make a set of equitation describing Ford Expedition model:

For period corresponding to 1st generation:

$$\begin{cases} f_1(t) = e^{10.736t}, & -1 < t < 0 \\ f_2(t) = 45974 + 168550t, & 0 < t < 1 \\ f_3(t) = 10221 + 240056t - 35753t^2 & 1 < t < 7 \end{cases}$$

For period corresponding to 2nd generation:

$$\{f_3(t) = -211007 + 111294t - 8147.29t^2, 6 < t < 11$$

For period corresponding to 3rd generation:

$$\{f_3(t) = -590577 + 134158t - 6638t^2, \quad 10 < t < 14$$

We chose negative transition point value of exponential growth of the first generation as the first Expedition generation was put on sale about one year previously. Figure 1 shows various functions of maternity piece describing. Transition points are chosen also expertly.

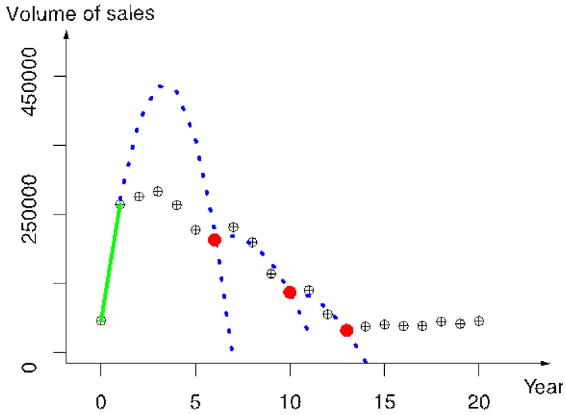


Figure 1: Sales volume modeling with innovation curve.

Another example of undivided interval described time series is car sales volume changing. Collected data include Ford Expedition's minimal, average and maximal prices values. When describing average prices with innovation curve we receive (Figure 2):

For the Ford Expedition 1st generation period:

Minimal price:

$$\begin{cases} f_1(t) = 27620 + 867.5t, & 0 < t < 2 \\ f_2(t) = 27368.8 + 1118.8t - 62.8t^2, & 2 < t < 6 \end{cases}$$

Average price:

$$\begin{cases} f_1(t) = 30914 + 1537.5t, & 0 < t < 2 \\ f_2(t) = 30070 + 2381.5t - 211t^2, & 2 < t < 6 \end{cases}$$

Maximal price:

$$\begin{cases} f_1(t) = 34225 + 2435t, & 0 < t < 2 \\ f_2(t) = 32406 + 4254t - 455t^2, & 2 < t < 6 \end{cases}$$

For the Ford Expedition 2nd generation period:

Minimal price:

$$\begin{cases} f_1(t) = 26915 + 817.5t, & 6 < t < 8 \\ f_2(t) = -66605 + 24198t - 1461.3t^2, & 8 < t < 10 \end{cases}$$

Average price:

$$\begin{cases} f_1(t) = 27943 + 1470t, & 6 < t < 8 \\ f_2(t) = -87225 + 30262t - 1799.5t^2, & 8 < t < 10 \end{cases}$$

Maximal price:

$$\begin{cases} f_1(t) = 27200 + 2390t, & 6 < t < 8 \\ f_2(t) = -150780 + 46890t - 2781.3t^2, & 8 < t < 10 \end{cases}$$

For the Ford Expedition 3rd generation period:

Minimal price:

$$\begin{cases} f_1(t) = 1245 + 2800t, & 10 < t < 12 \\ f_2(t) = -25233 + 7213t - 183.9t^2, & 12 < t < 19 \end{cases}$$

Average price:

$$\begin{cases} f_1(t) = 1690 + 3375.9t, & 10 < t < 12 \\ f_2(t) = -25528 + 7912t - 189t^2, & 12 < t < 19 \end{cases}$$

Maximal price:

$$\begin{cases} f_1(t) = 720 + 3927.5t, & 10 < t < 12 \\ f_2(t) = -34450 + 9789t - 244t^2, & 12 < t < 19 \end{cases}$$

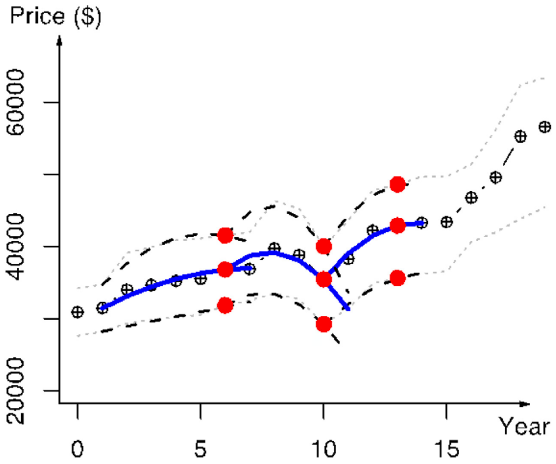


Figure 2: Price changing model described with innovation curve.

Knowledge of regularities in value changes makes it possible to describe them functionally and thereby forecast values necessary to plan economic activity and management decision making on the goods which they describe.

As a case in point, we compare the proposed describing method with functional describing with the method of least squares. Least-squares method applied to sales of Ford Expedition volume describing gives a function $y = 242800 - 5847x - 107x^2$, for the sales changing $y = 13707 - 1040x - 15x^2$, $y = 36487 - 909x - 62x^2$, $y = 38674 - 609x + 57x^2$ (for minimal, average and maximal prices accordingly).

Model verification with chi-square criterion for time series describing sales volume shows that LSM model does not pass inspection $(Y_i^э - \sigma \le Y_T \le Y_i^э + \sigma)$. Thus the model is not adequate. Increasing of polynomial order does not make any different basically. As is well known, considerable increasing of polynomial order decreases forecast precision in following periods. Model based on innovation curve passes inspection of chi-square criterion.

Both approaches give sales volume models that pass the chi-square criterion, but when the innovative curve is used we get a more accurate description.

As a result of conducted analysis, we could draw a conclusion that methods based on fitting criterion give the best fit to describe undivided time series when a large amount of statistic data are available. However in the case when a small amount of the data is available the knowledge of values changing regularities gives us better results. A short time period, which data could be forecasted, is a limitation of the method. It is related to the fact that we could forecast values of the unfinished time period, which is functionally describing.

To forecast long-term parameters knowledge of regularities of innovation curve time changing is necessary.

## 4 SPECIFICITY OF UNDIVIDED TIME SERIES FORECASTING HAVING INITIAL PART DESCRIBED WITH INNOVATION CURVE

The approach mentioned above could be applied when historical data are used. Due to the lack of data and resulting difficulty in practical forecasting, it is necessary to determine new parameters of the innovation curve on the basis of previous stages. The time series described with one of the innovation curve types could describe the life cycle of each product variety by means of the similar functions.

We could see confirmation of the hypothesis when dynamic time warping algorithm (Luzianin, 2016) is applied to maternity periods of car models (Figures 1 and 2). As a result, we could see that values' changing obeys the same tendencies, which could be represented as both displacement time and value axis and spread of the parabola's branches. The parabola branches behavior depends on parabola's factors. Unknown values could be found with axis displacement and parabola's branches spread estimation on basis of statistics. It allows extrapolating the tendencies to the future.

Quoted Ford Expedition statistics displays the tendency of sales volume decreasing and price increasing. This tendency could be evaluated through the axis displacement and parabola's branches spread. From previous computations, we got table 2. Formulas to complete the table are (Aufmann, 2008): $X_a = -\frac{b}{2a}$, $Y_a = -\frac{D}{4a}$, $D = b^2 - 4ac$ from the equation $y = ax^2 + bx + c$.

Table 2: Factors of parabola's displacement when functional sales volume and price changing describing of various Ford Expedition generations.

| | Vertex coordinates | | Focus coordinates ( $Y_f = Y_a - {}^1/_4 a$ ) | Vertex displacement | | Focus displacement |
|---|---|---|---|---|---|---|
| | $X_a$ | $Y_a$ | | $\Delta X$ | $\Delta Y$ | |
| Fig. 1 gen. 1 | 3,357 | 413172,4 | 422110,6 | — | — | — |
| Fig. 1 gen. 2 | 6,830 | 169068,9 | 171105,7 | 3,473 | -244103,5 | -251004,9 |
| Fig. 1 gen. 3 | 10,105 | 87276,6 | 88936,1 | 3,275 | -81792,3 | -82169,6 |
| Fig. 2 gen. 1 | 7,303 | 37000,7 | 37035,7 | — | — | — |
| Fig. 2 gen. 2 | 7,784 | 39193,5 | 39384,1 | 0,481 | 2192,7 | 2348,4 |
| Fig. 2 gen. 3 | 13,825 | 43267,3 | 43401,0 | 6,041 | 4073,9 | 4016,9 |

Table 2 shows that vertex and focus displacement could be described functionally (Figures 3 and 4). The Figure 3 shows that the parabola could be plotted with the derived points. It agrees with the assumption that sales volume decrease when price increasing.

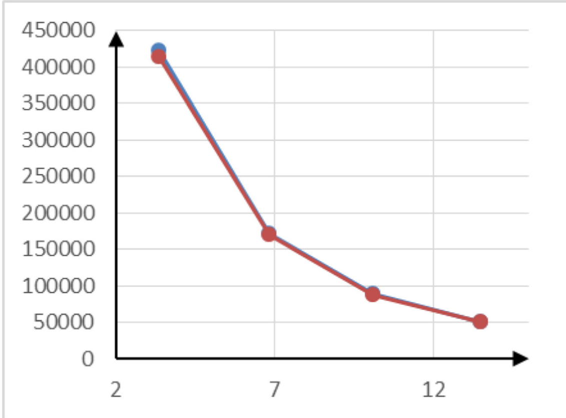


Figure 3: Value of the functions $Y_a(X_a)$ (the blue curve) and $Y_f(X_a)$ (the red curve) for sales volume changing.

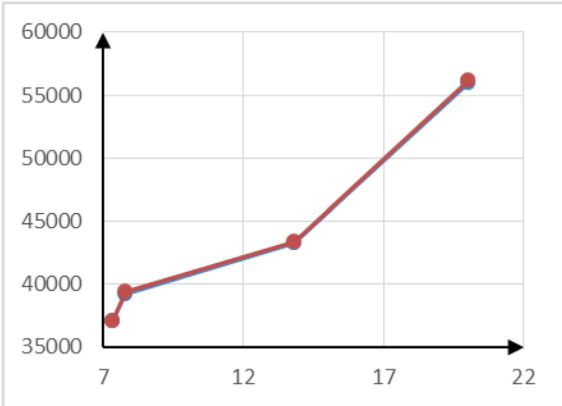


Figure 4: Value of the functions $Y_a(X_a)$ (the blue curve) and $Y_f(X_a)$ (the red curve) for price changing.

The fourth points were received for the forecasting curve.

We applied the received values to forecasting parabolic curves construction after the inverse factors determining. The results are shown in Figures 5 and 6. The received curves satisfy the model $\chi^2$ method verification.

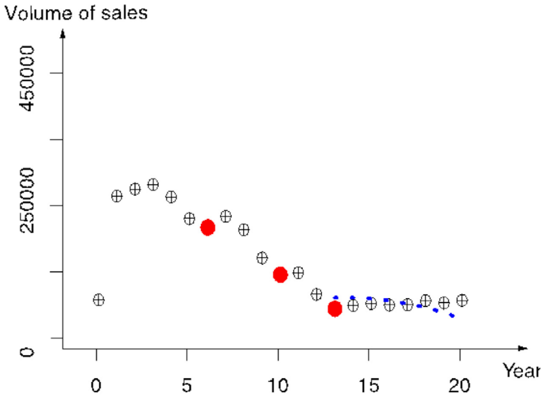


Figure 5: The fourth generation of Ford Expedition sales volume changing forecasting and its comparison to retrospective data.

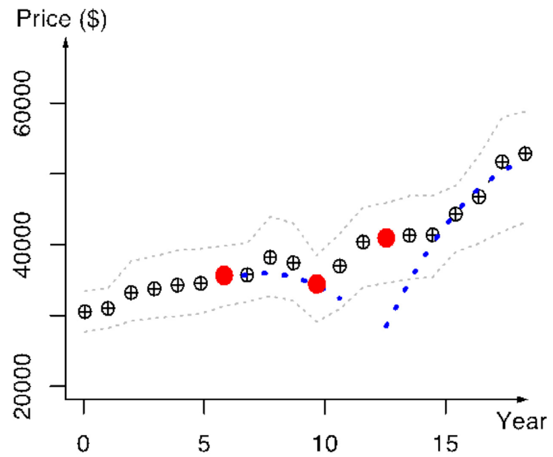


Figure 6: The fourth generation of Ford Expedition price changing forecasting and its comparison to retrospective data.

Another result is that selection of any value pairs doesn't change significantly type of the curves. And findings are also adequate. It allows making initial forecasts and updating when new data appearing. It could be made with both parabola vertex and focus coordinates specifying and e.g. factors determination with the least-square method. Moreover, received vertex coordinates of the parabola, which describes price changing of the Ford expedition the second generation, don't be on the curve (Figure 4). Vertex moving to the parabola gives us describing accuracy enhancement (Figure 6). It could be accounted in the case that the stated hypothesis about curves construction character is correct. The method of least squares allows only finding the optimum factors for available data.

To exclude random factor we made the same computation of the Ford Explorer model, which has also data on the several generations.

# 5 CONCLUSION

As a result of this investigation, the values describing sale volume and price changing on the periods next to the first one could be described with a parabolic function provided for enough statistics. Besides vertex and focus displacement also could be described mathematically. Thus, characteristics of new model output could be estimated before its production and specified with functional-analytic approach when the first statistics appearing.

Another result is that demand for any product without considerable modification which puts a new innovation curve will decrease when price increasing.

Dynamic time warping allows defining tendency and describing regularity of product parameters changing. However, the algorithm ignores model individuality. In particular, sales volume jumping distorts the forecasting displacement function.

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