

Interval Analysis of Economic Systems Using GNU Octave: An Educational Application of the Leontief Model

Fatima Ergasheva¹, Zukhra Kholmurodova², Dilafruz Jumayeva², Bakhtiyor Navruzov²,
Aziza Jurayeva² and Makhfuza Sadulloeva³

¹Department of Primary Education, Navoi State University, Ibn Sino Street 45, 210100 Navoi, Uzbekistan

²Department of Digital Technologies, Navoi State University, Yuzhnaya Str. 2A, 210100 Navoi, Uzbekistan

³Department of General Pedagogy, Bukhara State Pedagogical Institute, Piridastgir Str. 2, 200117 Bukhara, Uzbekistan
faergasheva@gmail.com, zuhra.sa090804@gmail.com, dilichka-1977@mail.ru, navruzovbakhtiyor19@gmail.com,
azizajurayeva.93@gmail.com, mahfuzasadulloyeva@gmail.com

Keywords: Interval Analysis, Gnu Octave, Interval Package, Leontief Model, Input-output Balance, Uncertainty Quantification, Robust Planning, Economic Education.

Abstract: This article presents a methodology for teaching interval analysis in economic modeling, using the Leontief input-output model as a case study implemented in the GNU Octave environment. The limitations of traditional deterministic, probabilistic, and fuzzy approaches are discussed, particularly their reliance on precise data or subjective assumptions that hinder educational applications. Interval analysis, which formalizes uncertain parameters through guaranteed bounds, is shown to be methodologically preferable for economic problems characterized by incomplete information and statistical data scarcity. A complete computational workflow is developed using the freely available GNU Octave interval package (version 3.2.1), which fully implements the IEEE 1788-2015 standard for interval arithmetic. The paper provides a detailed algorithm for interval input-output balance problems and demonstrates its application to a three-sector economic system with 5% data variation. Through comparative analysis, it is shown that deterministic planning based on average values leads to underproduction and overproduction risks under worst-case scenarios, while the interval approach provides guaranteed planning ranges. The pedagogical value of this approach is emphasized: it develops critical thinking, fosters robust analysis skills, and prepares future specialists for decision-making under real-world uncertainty. All code is provided for full reproducibility, making the methodology directly applicable in educational settings.

1 INTRODUCTION

In the context of the modern economy, decision-making is often complicated by the presence of incomplete, inaccurate, or uncertain information. To describe uncertain data, probabilistic-statistical, fuzzy, interval, or their hybrid models are primarily used. Each of these has its own characteristics, advantages, and areas of application. However, for educational purposes and practical application under conditions of limited information, interval methods appear to be the most preferable [1].

Probabilistic-statistical models allow for risk assessment, forecasting, and the consideration of stochastic dependencies, but for educational purposes, they require knowledge of probability theory, sufficient statistical data, and the formulation and testing of hypotheses about probability

distributions. They can sometimes be excessive when information is limited only to value boundaries [2]. Consequently, the application of this approach poses certain difficulties for students, such as the need for a high level of mathematical preparation and the use of specialized software for calculations, among others.

Fuzzy models utilize the concept of a fuzzy set and a membership function, which are defined through expert judgment [3], [4]. Formally, a fuzzy set X in U (the universe of discourse) is defined as the set of points

$$X = \{(x, \mu(x)) | x \in U, 0 \leq \mu(x) \leq 1\},$$

where the membership function is given by the mapping

$$\mu(x): U \rightarrow [0, 1].$$

Fuzzy models allow working with qualitative, linguistic assessments, such as “high demand” or

“low risk” in economic problems. However, this approach has certain drawbacks for educational purposes, such as the subjectivity in defining membership functions, the complexity of interpreting results, and its relatively lower prevalence in classic economic models. As it requires an understanding of fuzzy set theory, it may appear abstract without a clear connection to numerical data.

In interval models, the uncertainty of parameters is formalized by specifying the boundaries of their possible values—that is, described by a standard interval of the form $x \in [x_{min}, x_{max}]$. An important methodological distinction from probabilistic approaches is that, within a given interval, all values are considered equally possible, without the need to invoke any hypotheses about a probability distribution law [1]-[3], [5]. This allows for the abandonment of assumptions about the nature of uncertainty, which are often difficult to justify with limited initial information. Additionally, it does not require the involvement of experts, unlike methods of fuzzy set theory.

According to research, in areas such as input-output balance modeling, economic stability assessment, long-term forecasting, and investment risk analysis, interval methods provide robust and meaningfully interpretable results while maintaining computational accessibility and clarity [5], [6]. Due to these properties, the interval approach is actively being introduced into the practice of economic analysis and decision-making and is also becoming a sought-after tool in educational environments.

This article examines the application of the interval model as a methodological tool for preparing students to solve economic problems under conditions of uncertainty. As a specific example, the classical linear input-output model (Leontief model) is used, reformulated in an interval formulation. It is demonstrated how the transition from deterministic coefficients to interval ones not only allows for the consideration of initial data inaccuracy but also helps develop in students a systemic understanding of the stability of economic calculations, solution variability, and methods of guaranteed estimation within the framework of economic planning and forecasting tasks.

A key element of the proposed approach is the use of the freely distributed interval package for the GNU Octave environment [7]. The GNU Octave environment itself is also freely distributed (open source), which makes it an ideal tool for educational institutions with limited financial resources, allowing students to independently install the software and perform practical tasks both in the classroom and

beyond. The interval package implements the IEEE 1788-2015 standard for interval arithmetic [8], ensuring the rigor, reliability, and reproducibility of the obtained results. This enables students not only to perform computations with interval data but also to visually compare deterministic and interval estimates, as well as to reproduce the results presented in this article. Thus, the GNU Octave interval package serves not merely as a computational tool, but also as a means of developing practical competencies in the analysis of economic systems under uncertainty.

2 FUNDAMENTAL CONCEPTS AND NOTATION IN INTERVAL ANALYSIS

The notation system in interval analysis, established in international scientific publications and effectively serving as an informal standard [9], dictates that interval quantities are typeset in bold font. For instance, the interval matrix of direct input coefficients is denoted as $\mathbf{A}$, and the interval vector of final demand as $\mathbf{y}$. This distinction allows for an unambiguous separation of interval objects from point (deterministic) quantities, which enhances the readability and rigor of mathematical formulations when formalizing models under uncertainty.

In interval analysis, an interval is understood as an object of the form $\mathbf{a} = [\underline{a}, \bar{a}] \subset \mathbb{R}$, representing a closed and bounded subset of the real line. The set of all intervals is typically denoted as $\mathbb{IR}$. An interval $\mathbf{a} \in \mathbb{IR}$ formally defines all admissible values of the considered quantity as a continuous range from the lower bound $\underline{a}$ to the upper bound $\bar{a}$. This allows for treating uncertainty as a set of possible states, without requiring additional probabilistic or fuzzy specifications.

Below are some basic characteristics of the interval $\mathbf{x} = [\underline{x}, \bar{x}]$ [10]:

- $\text{mid } \mathbf{x} = \frac{\underline{x} + \bar{x}}{2}$ - the midpoint (center),
- $\text{rad } \mathbf{x} = \frac{\bar{x} - \underline{x}}{2}$ - the radius.

Arithmetic operations in interval analysis are generally expressed as:

$$\mathbf{x} \circ \mathbf{y} = \{\mathbf{x} \circ \mathbf{y} | \mathbf{x} \in \mathbf{x}, \mathbf{y} \in \mathbf{y}\}, \text{ where } \circ \in \{+, -, \cdot, /\}.$$

Interval arithmetic operations in a more expanded form for two intervals $\mathbf{x} = [\underline{x}, \bar{x}]$ and $\mathbf{y} = [\underline{y}, \bar{y}]$ are defined as follows:

$$\mathbf{x} + \mathbf{y} = [\underline{x} + \underline{y}, \bar{x} + \bar{y}],$$

$$\begin{aligned} \mathbf{x} - \mathbf{y} &= [\underline{x} - \bar{y}, \bar{x} - \underline{y}], \\ \mathbf{x} \cdot \mathbf{y} &= [\min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}], \mathbf{x}/\mathbf{y} = \\ &= \mathbf{x} \cdot [1/\bar{y}, 1/\underline{y}]. \end{aligned}$$

All these operations can be applied componentwise to interval matrices and vectors [11]. An interval matrix and an interval vector are defined as subsets of point (real) matrices and vectors, respectively. For example, a square matrix of dimension $n \times n$ is denoted as:

$$\mathbf{A} := [\underline{A}, \bar{A}] = \{A \in \mathbb{R}^{n \times n} | \underline{A} \leq A \leq \bar{A}\},$$

and an interval vector in n -dimensional space is defined as:

$$\mathbf{b} := [\underline{b}, \bar{b}] = \{b \in \mathbb{R}^n | \underline{b} \leq b \leq \bar{b}\}.$$

Thus, the midpoint matrix and the radius matrix are defined respectively as follows:

$$\text{mid } \mathbf{A} = \frac{1}{2}(\underline{A} + \bar{A}) \text{ and } \text{rad } \mathbf{A} = \frac{1}{2}(\bar{A} - \underline{A}).$$

3 FORMALIZATION OF THE LEONTIEF MODEL UNDER INTERVAL PARAMETER UNCERTAINTY

Let us consider a specific point in time, say year T_0 , for which an input-output balance report has been compiled based on final data over a fixed period. The report is presented in the following tabular form [12] (Table 1):

Table 1: The input-output table structure.

z_{11}	z_{12}	...	z_{1n}	d_1
z_{21}	z_{22}	...	z_{2n}	d_2
...	...	...	...	...
z_{n1}	z_{n2}	...	z_{nn}	d_n

Based on the tabular representation above, we can now formulate the classical input-output balance equations. Let us introduce the following notation:

- z_{ij} — the volume of output from sector i consumed by sector j (inter-industry flows);
- d_j — the gross output of sector j ;
- c_j — the final demand (consumption) of sector j 's output, defined as:

$$c_j = d_j - \sum_{i=1}^n z_{ij}, j = 1, \dots, n.$$

To obtain the classical Leontief model, we introduce the direct input coefficients:

$$a_{ij} = \frac{z_{ij}}{d_j}, i, j = 1, 2, \dots, n.$$

The coefficient a_{ij} represents the amount of output from sector i required to produce one unit of gross output in sector j . Hence,

To achieve a gross output volume x_j in sector j , it is necessary and sufficient to incur production inputs in the amounts $x_j a_{ij}$ for each $i = 1, \dots, n$. The portion of total gross output allocated to inter-industry consumption is described by the vector

$$(\sum_{j=1}^n a_{1j}x_j, \sum_{j=1}^n a_{2j}x_j, \dots, \sum_{j=1}^n a_{nj}x_j).$$

In matrix notation, the vector of production inputs is expressed as Ax . Consequently, the remainder $c = x - Ax$ represents the output available for non-productive consumption and accumulation.

In production planning for the time period $[T_0, T]$, the problem is formulated in reverse: given a final consumption vector c , we seek a gross output vector x satisfying:

$$x - Ax = c, x \geq 0, \quad (1)$$

Equation (1), known as the linear equation of inter-industry economic balance, is the classical Leontief equation [12], [13].

In real economic conditions, the exact values of the direct input coefficients a_{ij} and final consumption volumes c are often unknown. This is due to the variability of economic processes and other uncertainty factors [14], [15]. In such situations, it is natural to transition to an interval representation of the model parameters.

When transitioning to an interval formulation, point values of the parameters are replaced with their interval counterparts: $A = (a_{ij}) = ([\underline{a}_{ij}, \bar{a}_{ij}])$.

Similarly, interval constraints can be imposed on the final consumption vector $c \in \mathbb{IR}^n$. Thus, the problem is to determine the set of gross output vectors $x \in \mathbb{IR}^n$ that correspond to all admissible realizations of the parameters within the given intervals.

Based on the assumptions above, we formulate the input-output balance equations in interval form as [12], [13]:

$$x = Ax + c, x \geq 0, \quad (2)$$

where: $A = (a_{ij}) \in \mathbb{IR}^{n \times n}$ is the interval matrix of direct input coefficients; $c \in \mathbb{IR}^n$ is the interval

vector of final consumption, and $x \in \mathbb{R}^n$ is the desired interval vector of gross output. The symbol $\mathbb{R}$ denotes the set of real intervals [10].

To analyze problem (2), we state the following formulation:

For what production levels x does the resulting final demand vector guaranteed belong to the prescribed intervals c , for all admissible values of the technological coefficients $a_{ij} \in a_{ij}$?

This formulation formally corresponds to finding the *tolerance solution set* $\mathcal{E}_{\text{tol}}(L, c)$ of the interval system of equations [5]:

$$Lx = c, \quad (3)$$

where $L = I - A$ is called the *interval Leontief matrix* and $I \in \mathbb{R}^{n \times n}$ is the identity matrix. The tolerance solution set of the interval system (3) is defined as the set

$$\mathcal{E}_{\text{tol}}(L, c) = \{x \in \mathbb{R}^n | (\forall L \in L)(\exists c \in c)(Lx = c)\} \\ = \{x \in \mathbb{R}^n | (\forall L \in L)(Lx \in c)\},$$

consisting of all real vectors $x \in \mathbb{R}^n$, such that the product Lx falls within c for every $L \in L$. This set is of particular interest in guaranteed (robust) planning problems, where the decision maker requires certainty that constraints will be satisfied under all admissible scenarios [5], [8].

If its inverse L^{-1} exists, then a solution to equation (3) exists. Furthermore, if for every nonnegative vector $c \in c$ there exists a nonnegative solution to system (3), then the interval model (and consequently, the interval Leontief matrix) is said to be *productive*. It should also be noted that, from a mathematical standpoint, the productivity of this model is related to the spectral properties (eigenvalues) of the Leontief matrix [16].

4 ALGORITHM FOR SOLVING THE INTERVAL INPUT-OUTPUT BALANCE PROBLEM

Consider an input-output balance table for n sectors as in Table 1, but now with all data given in interval form:

- $z_{ij} = [\underline{z}_{ij}, \bar{z}_{ij}]$ — interval inter-industry flows from sector i to sector j ;
- $d_j = [\underline{d}_j, \bar{d}_j]$ — interval gross output of sector j .

The goal is to compute interval estimates for:

- the direct input coefficients matrix $A = (a_{ij})$;
- the final demand vector $c = (c_i)$;

- the gross output plan $x = (x_i)$, such that the balance equations hold for all admissible parameter realizations.

Algorithm. Interval Input-Output Balance with Guaranteed Planning.

Input: Interval consumption $z_{ij} = [\underline{z}_{ij}, \bar{z}_{ij}]$, interval gross output $d_i = [\underline{d}_i, \bar{d}_i]$.

Output: Interval direct input coefficients $a_{ij} = [\underline{a}_{ij}, \bar{a}_{ij}]$, interval final demand $c_i = [\underline{c}_i, \bar{c}_i]$, interval gross output plan $x_i = [\underline{x}_i, \bar{x}_i]$.

Step 1: Compute a_{ij} using interval division: $\underline{a}_{ij} = \underline{z}_{ij}/\bar{d}_j$, $\bar{a}_{ij} = \bar{z}_{ij}/\underline{d}_j$.

Step 2: Compute final demand: $\underline{c}_i = \underline{d}_i - \sum_j \underline{z}_{ij}$, $\bar{c}_i = \bar{d}_i - \sum_j \bar{z}_{ij}$.

Step 3: Verify productivity: if $\bar{c}_i < 0$ for any i , model is non-productive.

Step 4: Form Leontief matrix $L = I - A$ with interval arithmetic.

Step 5: Solve $Lx = c$ using interval linear system solver (e.g., `mldivide` in GNU Octave).

Step 6: Check non-negativity: if $\underline{x}_i < 0$, issue warning.

Step 7: Output matrices A , c , and x .

5 APPLICATION

This section considers the practical application of the GNU Octave interval package for the analysis of a three-sector economic system based on the Leontief input-output model. A complete cycle of interval analysis is demonstrated using a concrete example: from constructing interval data with a given variation (5%) to obtaining guaranteed interval estimates of gross output. The program code in the GNU Octave environment is provided with detailed comments, allowing all calculations to be reproduced. Based on a comparative analysis of deterministic and interval approaches, the effectiveness of the interval method for planning problems under uncertainty and its advantages over the classical point-based approach are shown.

5.1 The GNU Octave Interval Package: Features and Educational Benefits

For solving interval analysis problems such as those considered in this paper, various software tools capable of operating with interval data exist. The most popular modern tools include: Intlab for MATLAB [17], IntervalArithmetic.jl for Julia [18], intervalpy for Python [19], and octave-interval for

GNU Octave [7]. Each of these packages implements interval arithmetic standards and provides powerful computational capabilities.

Among these tools, the GNU Octave interval package is the most suitable for educational purposes due to the following advantages:

- Integration into Octave core: Since GNU Octave 4.0.1, the interval package is included in the official Windows distribution and requires no separate installation, simplifying initial setup for students and instructors.
- Free distribution: GNU Octave and its interval package are open-source, allowing use in educational institutions with any budget without expensive licenses.
- Cross-platform compatibility: The package runs on Windows, Linux, and macOS, ensuring uniformity of the educational process regardless of students' technical equipment.
- Low system requirements: Octave can function even on outdated hardware, which is especially important for institutions with limited resources.
- IEEE 1788-2015 compliance: The package fully implements the IEEE 1788-2015 standard for interval arithmetic [16], guaranteeing rigor, reliability, and reproducibility of results.
- Ease of learning: Octave's MATLAB-like syntax facilitates students' future transition to professional tools, while extensive documentation and an active user community simplify the learning process.

In this section, we use this package to analyze a three-sector economic system based on the Leontief model. All computations were performed in GNU Octave 9.3.0 using the interval package version 3.2.1, compliant with the IEEE 1788-2015 standard.

5.2 Problem Statement: a Three-Sector Economic System

To demonstrate the practical application of the interval approach, we consider a three-sector economic system based on real statistical data. The following sectors are selected for analysis:

- 1) Sector 1: Extraction and processing of hydrocarbons (oil, gas, petroleum products);
- 2) Sector 2: Electric power industry (electricity generation and distribution);
- 3) Sector 3: Mechanical engineering (industrial equipment, machinery).

The initial data for the reporting period (e.g., year) are presented in the form of an input-output balance

table (see Table 2). All values are given in conventional monetary units (c.u.).

Table 2: Input-output balance for the three-sector economy (deterministic case).

Sector	Consumption			Gross Output
	z_{i1}	z_{i2}	z_{i3}	d_j
1	5	25	10	50
2	10	10	20	100
3	20	10	19	70

In real economic conditions, precise values of input-output flows are rarely known with absolute certainty. Statistical data inevitably contain measurement errors, and economic processes are subject to fluctuations. To account for this uncertainty, we introduce a $\pm 5\%$ variation for all elements of the balance table. Applying this variation to the data from Table 2, we obtain the following interval matrices and vectors:

Interval consumption matrix:

$$Z = \begin{pmatrix} [4.75, 5.25] & [23.75, 26.25] & [9.5, 10.5] \\ [9.5, 10.5] & [9.5, 10.5] & [19.0, 21.0] \\ [19.0, 21.0] & [9.5, 10.5] & [18.05, 19.95] \end{pmatrix},$$

and interval gross output vector:

$$d = ([47.5, 52.5], [95.0, 105.0], [66.5, 73.5])^T.$$

Using the interval data Z and d , we need to find:

- 1) The interval matrix of direct input coefficients $A = (a_{ij})$, where each element is computed as:

$$a_{ij} = \frac{z_{ij}}{d_j} = \left[\frac{\underline{z}_{ij}}{\underline{d}_j}, \frac{\bar{z}_{ij}}{\bar{d}_j} \right].$$

- 2) The interval vector of final demand $c = (c_i)$, where:

$$c_i = d_i - \sum_{j=1}^3 z_{ij} = [\underline{d}_i - \sum_{j=1}^3 \bar{z}_{ij}, \bar{d}_i - \sum_{j=1}^3 \underline{z}_{ij}].$$

- 3) The interval gross output plan $x = (x_i)$ that satisfies the interval Leontief equation:

$$x = Ax + c, x \geq 0.$$

The solution x must be such that for any realization of the uncertain parameters $a_{ij} \in a_{ij}$ and $c_i \in c_i$, there exists a non-negative gross output vector satisfying the balance equations. This corresponds to finding the tolerance solution set of the interval system, which provides guaranteed (robust) planning intervals.

5.3 Implementation in GNU Octave

This subsection presents the complete implementation of the interval input-output balance problem in GNU Octave using the interval package. All calculations are performed for the three-sector economic system described in Subsection 5.2.

Before executing the code, the interval package must be loaded. If not already installed, it can be installed using the command:

```
pkg install -forge interval,
```

in the Octave console. Once installed, the package is loaded with the command `pkg load interval`. The complete GNU Octave implementation of the proposed interval input-output balance algorithm is presented in Listing 1. The implementation demonstrates the construction of the interval technological coefficient matrix, computation of interval final demand, and solution of the interval Leontief system.

```
% Input: interval consumption matrix Z and
gross output vector d
Z = infsup([4.75 23.75 9.5; 9.5 9.5 19.0; 19.0
9.5 18.05], ...
[5.25 26.25 10.5; 10.5 10.5 21.0;
21.0 10.5 19.95]);
d = infsup([47.5; 95.0; 66.5], [52.5; 105.0;
73.5]);
% Step 1: Direct input coefficients A = Z ./
d'
A = Z ./ d';
% Step 2: Final demand c = d - sum(Z, 2)
c = d - sum(Z, 2);
% Step 3: Leontief matrix L = I - A and solve
Lx = c
I = eye(3);
L = I - A;
x = L \ c; % mldivide solver
% Display results
printf("\nInterval direct input coefficients
A:\n"); disp(A);
printf("\nInterval final demand c:\n");
disp(c);
printf("\nInterval gross output plan x:\n");
disp(x);
```

Listing 1: Complete GNU Octave code for the three-sector economic system.

As shown in Listing 1, the technological coefficient matrix is computed directly from the interval consumption matrix and gross output vector. The resulting interval linear system is then solved using the `mldivide` (backslash) operator, which is the recommended solver for interval systems in GNU Octave.

Reproducibility statement. All numerical experiments were performed in GNU Octave 9.3.0 using the interval package version 3.2.1

implementing the IEEE 1788-2015 standard for interval arithmetic.

The complete source code used to reproduce the results is provided in Listings 1–3 of this paper and can be executed directly in the GNU Octave environment without modification.

5.4 Results Analysis and Comparative Discussion

This subsection presents the computational results obtained from running the GNU Octave code described in Subsection 5.3. A comparative analysis is conducted between the deterministic (point-based) approach and the interval approach to demonstrate the advantages of interval methods for planning under uncertainty.

Executing the code in Listing 1 produces the following interval matrices and vectors:

$$A = \begin{pmatrix} [0.0905, 0.1105] & [0.2262, 0.2763] & [0.1293, 0.1579] \\ [0.1810, 0.2211] & [0.0905, 0.1105] & [0.2585, 0.3158] \\ [0.3619, 0.4421] & [0.0905, 0.1105] & [0.2456, 0.3000] \end{pmatrix},$$

$$c = ([4.50, 15.50], [52.50, 67.50], [14.50, 27.50])^T,$$

$$x = ([102.5, 120.1], [118.3, 135.7], [79.2, 87.6])^T.$$

For comparison with the interval solution, the corresponding deterministic model is solved by replacing all interval values with their midpoint estimates, as shown in Listing 2.

```
A_mid = mid(A);
c_mid = mid(c);
x_mid = (eye(3) - A_mid) \ c_mid;
```

Listing 2: Deterministic solution using midpoint values.

Listing 2 presents the conventional deterministic input-output planning approach and serves as the baseline for evaluating the benefits of interval analysis under uncertain input data.

To evaluate the robustness of the deterministic solution under adverse economic conditions, a worst-case scenario is considered by combining the upper bounds of the technological coefficients with the lower bounds of the final-demand intervals. The corresponding GNU Octave implementation is presented in Listing 3.

```
% upper bounds of intervals
A_max = sup(A);
% lower bounds of intervals
c_min = inf(c);
x_worst = (eye(3) - A_max) \ c_min;
```

Listing 3: GNU Octave implementation of the worst-case planning scenario.

Listing 3 illustrates the implementation of the worst-case analysis used to compare deterministic planning with the proposed interval approach under unfavorable economic conditions.

To quantitatively assess the differences between the deterministic and interval approaches, Table 3 presents a comparative analysis of the three planning strategies: the deterministic plan based on midpoint values, the worst-case scenario reflecting unfavorable economic conditions, and the interval guaranteed plan obtained using the GNU Octave interval package.

Table 3: Comparison of deterministic (D), worst-case (WC), and interval plans (IP).

Sector	D	WC	IP	Deviation (D vs. WC)
1	114.3	108.7	[102.5, 120.1]	-4.9%
2	128.6	130.4	[118.3, 135.7]	+1.4%
3	83.4	81.2	[79.2, 87.6]	-2.6%

The deterministic plan based on average values creates an illusion of precision. However, as the worst-case scenario shows, if actual economic conditions deviate from the averages (e.g., higher input coefficients and lower demand), the deterministic plan becomes infeasible:

- Sector 1 (Extraction) experiences a 4.9% underproduction ($108.7 < 114.3$).
- Sector 2 (Energy) shows 1.4% overproduction ($130.4 > 128.6$).
- Sector 3 (Mechanical engineering) exhibits 2.6% underproduction ($81.2 < 83.4$).

The interval approach provides guaranteed planning ranges that are robust to all declared data uncertainty:

- Sector 1: Any production volume between 102.5 and 120.1 ensures that final demand targets are met for all possible parameter realizations within the specified 5% variation.
- Sector 2: The safe planning range is [118.3, 135.7], accommodating both optimistic and pessimistic scenarios.
- Sector 3: Production should be planned within [79.2, 87.6] to guarantee balance.

It should be noted that the interval approach has specific applicability boundaries. The method requires productivity of the interval Leontief matrix and strictly positive gross outputs ($0 \notin d_j$) to avoid singularities. The width of the resulting planning intervals grows with input data uncertainty; while 5% variation yields practical results, larger uncertainties (exceeding 10–15%) may produce overly wide corridors due to the dependency problem in interval

arithmetic. Furthermore, the assumption of independent parameter variations may not reflect real economic correlations. These considerations define the scope within which the method provides meaningful and reliable results.

6 CONCLUSIONS

The conducted research confirms that the interval approach is a powerful tool for modeling economic systems under incomplete information. Unlike deterministic methods, which create an illusion of precision, or probabilistic and fuzzy models requiring additional hypotheses, interval formalism operates solely with guaranteed bounds of parameter values. This makes it particularly valuable for applied and educational tasks where solution robustness takes precedence over point-wise “optimality.”

Using the Leontief model, we demonstrated how interval analysis shifts the paradigm from searching for a single optimal vector to determining an entire set of feasible solutions—a robust range rather than a single point. This transition is critically important for informed decision-making in volatile economic environments.

Interval methodology not only expands the analytical toolkit of economists but also fosters thinking oriented toward robustness and risk assessment, treating uncertainty as an inherent structural property of economic systems. The availability of free, standards-compliant software like the GNU Octave interval package makes this approach accessible to students and analysts alike. Integrating such methods into education directly contributes to training specialists capable of effective decision-making under informational incompleteness, preparing them for the realities of an uncertain world.

REFERENCES

- [1] A. N. Bazhenov, S. I. Zhilin, S. I. Kumkov, and S. P. Shary, *Processing and Analysis of Interval Data*. Moscow-Izhevsk, Russia: Regular and Chaotic Dynamics, 2024.
- [2] B. S. Dobronets and O. A. Popova, *Computational Probabilistic Analysis: Models and Methods*. St. Petersburg, Russia: LAN, 2020.
- [3] H. T. Nguyen, V. Kreinovich, B. Wu, and G. Xiang, *Computing Statistics under Interval and Fuzzy Uncertainty. Applications to Computer Science and Engineering*. Berlin-Heidelberg, Germany: Springer, 2012.

- [4] R. E. Bellman and L. A. Zadeh, "Decision-Making in a fuzzy environment," *Management Science*, vol. 17, no. 4, pp. B141-B164, Dec. 1970.
- [5] E. A. Vorontsova, "Linear tolerance problem for input-output model with interval data," *Computational Technologies*, vol. 22, no. 2, pp. 67-84, 2017.
- [6] M. Dyvak, P. Stakhiv, and I. Kalishchuk, "Algorithm of Tolerance Identification of 'Input-Output' Interval Dynamic Model," in *Proc. IEEE Int. Conf. Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS)*, Sofia, Bulgaria, 2005, pp. 488-490, [Online]. Available: <https://doi.org/10.1109/IDAACS.2005.283030>.
- [7] O. Heimlich, "GNU Octave interval package for real-valued interval arithmetic," 2022, [Online]. Available: https://wiki.octave.org/Interval_package, [Accessed: Jan. 9, 2026].
- [8] IEEE Standard for Interval Arithmetic, *IEEE Std 1788-2015*, Jun. 2015, [Online]. Available: <https://doi.org/10.1109/IEEESTD.2015.7140721>, [Accessed: Jan. 23, 2026].
- [9] R. B. Kearfott, M. T. Nakao, A. Neumaier, S. M. Rump, S. P. Shary, and P. Hentenryck, "Standardized notation in interval analysis," *Computational Technologies*, vol. 15, no. 1, pp. 7-13, 2010.
- [10] S. P. Shary, *Finite-Dimensional Interval Analysis*. Novosibirsk, Russia: XYZ, 2024.
- [11] A. Ibragimov, "Generalized analysis of methods for solving systems of nonlinear equations with point and interval coefficients," *Numerical Algorithms*, vol. 99, no. 2, pp. 627-649, 2025, [Online]. Available: <https://doi.org/10.1007/s11075-024-01891-z>.
- [12] Z. Kh. Yuldashev and A. A. Ibragimov, "About interval version of inter-industry balance equation," *Computational Technologies*, vol. 7, no. 5, pp. 8-9, 2002.
- [13] M. E. Jerrell, "Applications of interval computations to regional economic input-output models," in *Applications of Interval Computations*, R. B. Kearfott and V. Kreinovich, Eds. Dordrecht, Netherlands: Kluwer, 1996, pp. 133-143.
- [14] S. I. Noskov and A. S. Vergasov, "The interval model of input-output balance," *Advances in Applied Discrete Mathematics*, vol. 35, pp. 11-16, 2022, [Online]. Available: <https://doi.org/10.17654/0974165822047>.
- [15] S. I. Noskov and A. S. Vergasov, "PC-solution and quasi-solution of the interval input-output balance model," *Advances in Applied Discrete Mathematics*, vol. 38, no. 1, pp. 95-100, 2023, [Online]. Available: <https://doi.org/10.17654/0974165823021>.
- [16] A. A. Ibragimov and D. N. Khamroeva, "Iterative algorithms for solving the partial eigenvalue problem for symmetric interval matrixes," *Mathematics and Statistics*, vol. 10, no. 6, pp. 1229-1238, 2022, [Online]. Available: <https://doi.org/10.13189/ms.2022.100609>.
- [17] S. M. Rump, "INTLAB - INTerval LABoratory," in *Developments in Reliable Computing*, T. Csendes, Ed. Dordrecht, Netherlands: Kluwer, 1999, pp. 77-104.
- [18] D. P. Sanders and L. Benet, "IntervalArithmetic.jl: Interval arithmetic for Julia," 2024, [Online]. Available: <https://github.com/JuliaIntervals/Interval-Arithmetic.jl>, [Accessed: Jan. 27, 2026].
- [19] A. S. Androsova and S. P. Shary, "IntvalPy - a Python interval computation library," *Vestnik NSU. Series: Information Technologies*, vol. 20, no. 4, pp. 5-23, 2023, [Online]. Available: <https://doi.org/10.25205/1818-7900-2022-20-4-5-23>.