# Robust Assessment of the Degree of Influence of Identification Errors on the Dynamic Properties of a Process Model in a Multilevel Control System

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Keywords: Multilevel Dynamic Object, Impact Assessment, Dynamic Properties, Identification Error, Unmeasurable Disturbance, Model Sensitivity.

Abstract: Assessing the impact of identification errors on the dynamic properties of multi-level objects under unmeasured disturbances involves analyzing deviations in model parameters and their impact on the system's stability and accuracy. Errors at a single level (e.g., the lower control level) can cascade down the dynamics of the entire multi-level structure, especially if the disturbance cannot be measured and compensated for directly. This article analyzes the impact of identification errors on a model obtained using the least-squares method. A model of statistical errors in parameter estimation is proposed, and a probabilistically representative set of process models is defined. The estimated parameter vector is significantly amplified by small singular values, making the least-squares method extremely unreliable and impossible to obtain an accurate parameter estimate. To improve the stability of the estimate, Tikhonov's regularization method was used, which adds a robust functional constraint to the classical least-squares correction criterion and introduces a regularization factor to regulate the balance. The results of this study, i.e., an assessment of the dynamic characteristics of the process model using a representative set, allow us to determine the upper limit of the amplitude-frequency response and obtain an estimate of the lower limit.

## 1 INTRODUCTION

Literature analysis shows that in the presence of unmeasured disturbances, standard algorithms (e.g., least squares) produce biased estimates. This distorts the transfer functions and frequency characteristics of the model [1], [2]. Identification errors at different levels of the hierarchy accumulate, which can lead to a loss of stability in deviation-based control, especially if the plant does not have self-alignment properties [3]-[5]. For multi-level systems, the consistency of level models is critical. An error in the parameters of one level (e.g., due to external background or noise) causes a "cascading" deviation in the prediction of the behavior of the entire system [6], [7].

The following approaches exist to solve the above-mentioned problems: the use of algorithms that minimize parameter errors under stochastic noise conditions (e.g., Kalman filters or online error calculation algorithms) [1], [4], the development of observers to isolate low-frequency components of unmeasured disturbances, which allows separating the influence of the external environment from the dynamics of the object itself [2], [3], the use of finite-frequency identification to improve the accuracy of model parameters in the operating frequency range [6], [7], the use of binary or hybrid intelligent models that adapt to changing conditions and compensate for structural uncertainty [8]-[10]. An analysis of the impact of identification errors on the model obtained by the least squares method is presented below. The paper considers a model of statistical errors in parameter estimation and defines a probabilistic representative set of process models.

Let us consider the problem of identifying a process, which, using a dynamic representation, can be expressed in the form:

$$\begin{cases} y(kT^*) = U(kT^*)\Theta + \varepsilon(kT^*), \\ \Theta = [a_1, a_2, \dots, a_m]^T, \\ U(kT^*) = [u_1(kT^*), \dots, u_m(kT^*)], \end{cases} \quad (1)$$

where $y$ - is the output signal, $U$ is the vector of generalized inputs, $\varepsilon$ - unmeasurable disturbance, $kT^*$- the value of discrete time, $T^*$- sampling interval, and $\Theta$ - vector of estimated parameters.

It is assumed that the signals $y$, $U$, $\varepsilon$ are stationary, ergodic and have zero mathematical expectations ($a$), while the signals $U$ and $\varepsilon$ mutually independent ($b$); the structure of model (1) is specified correctly ($c$), and based on the signals $y$, $U$ using the least squares method, an unbiased estimate of the parameter vector of model (1) can be obtained ($d$), and the output error of the estimated model

$$e(kT^*) = y(kT^*) - \hat{y}(kT^*),$$

has the properties of white noise, where $\hat{y}(kT^*)$ - the model output ($e$), determined using the estimated parameter vector.

In applied IT terms, the proposed approach can be interpreted as a computational workflow for uncertainty-aware identification of multilevel control objects under unmeasured disturbances. The workflow combines parameter estimation, regularization, representative-set construction, and frequency-domain analysis to support robust digital decision-making in process control systems.

# 2 STATEMENT OF THE PROBLEM

The task consists of consistently defining a model that describes the errors in estimating the parameter vector, then constructing a representative set of models of the process under consideration that is achievable in a probabilistic sense and provides an adequate representation of the uncertainty of the original system, as well as in the subsequent analysis of frequency characteristics and the study of their extreme values for the moving average model and the autoregressive moving average model using the specified representative set of models.

## 2.1 Definition of a Model that Describes the Errors in Estimating a Vector of Parameters

Representation (1) can be expressed as a vector equation:

$$Y = \begin{bmatrix} y(T^*) \\ \vdots \\ y(MT^*) \end{bmatrix},$$

$$U = \begin{bmatrix} u_1(T^*) & \dots & u_m(T^*) \\ u_1(MT^*) & \dots & u_m(MT^*) \end{bmatrix},$$

$$\varepsilon = \begin{bmatrix} \varepsilon(T^*) \\ \vdots \\ \varepsilon(MT^*) \end{bmatrix}.$$

Least squares estimation of the model parameter vector:

$$\hat{Y} = U\hat{\Theta}, \quad (2)$$

where $\hat{Y} = [\hat{y}(T^*), \dots, \hat{y}(MT^*)]^T$, is defined as follows:

$$\begin{cases} \hat{\Theta} = (U^T U)^{-1} U^T Y, \\ \hat{\Theta} = [\hat{a}_1, \dots, \hat{a}_m]^T, \end{cases} \quad (3)$$

with an estimate of the variance of the model output error $\sigma_\varepsilon^2 \cong \frac{1}{M-m} e^T e$.

In [11], [12] it was established that solution (2) will be stable if:

$$\kappa(U)\left(\varepsilon_c + \frac{1}{\tau}\right) << 1,$$

where $\varepsilon_c$ - computational error, and $\kappa(U)$ - condition number of the matrix $U$.

When the above assumptions ($a$)-($e$) are met, the error covariance matrix is expressed as:

$$cov(\hat{\Theta}) \cong (U^T U)^{-1} \sigma_\varepsilon^2.$$

Let us denote by $\tilde{u}_i(kT^*)$ - orthogonally transformed input signals:

$$\tilde{u}_i(kT^*) = u_i(kT^*) - \sum_{j=1}^{i-1} \alpha_j^i \tilde{u}_j(kT^*), \; i = 1, \dots, m,$$

where $\alpha_j^i = \frac{1}{M} \sum_{k=1}^{M} u_i(kT^*) \tilde{u}_j(kT^*)$.

Using transformed inputs $\tilde{u}_i$, model (2) can be represented as:

$$\hat{Y} = \tilde{U}\tilde{\Theta}, \qquad \tilde{\Theta} = [\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m]^T, \quad (4)$$

where $U = \tilde{U}A$,

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ \alpha_2^1 & 1 & \dots & 0 & 0 \\ \vdots & & & & \\ \alpha_m^1 & \alpha_m^2 & \dots & \alpha_m^{m-1} & 1 \end{bmatrix}.$$

At the same time:

$$\hat{\Theta} = A^{-1}\tilde{\Theta}. \quad (5)$$

Least squares estimation $\tilde{\Theta}$ using transformed inputs is determined by the expression:

$$\tilde{\Theta} = \left(\tilde{U}^T\tilde{U}\right)^{-1}\tilde{U}^T Y = \Lambda\tilde{U}^T Y,$$

where $\Lambda = diag[\lambda_1^{-1}, \lambda_2^{-1}, \dots, \lambda_m^{-1}]$, and:

$$\lambda_i = \sum_{k=1}^{M} u_j^2(kT^*) - \sum_{j=1}^{i-1}\left(\alpha_j^i\right)^2\lambda_j^2,\ i = 1, \dots, m.$$

In this case, the parameter error variance $\tilde{\Theta}$ is expressed as follows:

$$cov\left(\tilde{\Theta}\right) = \Lambda\sigma_e^2,$$

where the diagonal form of the covariance matrix means that each of the parameters $\tilde{a}_i$ is determined with an error whose variance is equal to $\sigma_\varepsilon^2/\lambda_i$, and the errors are independent. In accordance with the above assumptions (*a*), (*b*), we can represent the vector $\tilde{\Theta}$ parameters $\tilde{a}_i$ in the form of a multidimensional stochastic process

$$\sigma = [\sigma_1, \sigma_2, \dots, \sigma_m]^T, \quad (6)$$

with a normal distribution when $M \to \infty$, mathematical expectations $E[\sigma_i] = \tilde{a}_i$ and covariances:

$$\begin{cases} E[(\sigma_i - \tilde{a}_i)^2] = \delta_1^2 = \sigma_\varepsilon^2/\lambda_i\,, \\ E\left[(\sigma_i - \tilde{a}_i)\left(\sigma_j - \tilde{a}_j\right)\right] = 0, i \neq j, i, j = 1, \dots, m. \end{cases} \quad (7)$$

Vector value $\hat{\Theta}$ in (3) can be expressed using (5):

$$\hat{\Theta} = A^{-1}\sigma.$$

Hence:

$$E\left[\hat{\theta}\right] = A^{-1}E[\delta] = A^{-1}\tilde{\Theta},$$
$$cov\left[\hat{\theta}\right] = A^{-1}\Lambda(A^{-1})^T\sigma_\varepsilon^2,$$

and the error in estimating the parameter vector $\hat{\Theta}$ can be considered as a multivariate normally distributed process with parameters determined by expressions (7).

## 2.2 A regularization Method for Increasing the Stability of the Estimation of the Model Parameter Vector

The coefficients of the characteristic polynomial are determined from the system of equations (2), in which the matrix $U$ and the right-hand side are approximately given $\hat{Y}$. This problem, even for small dimensions, becomes significantly ill-posed, since the condition number of the matrix $U$ grows very rapidly with increasing its dimension [11]-[13].

In equation (2) $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq 0$- singular values of the matrix $U$. If the equation is ill-conditioned, then the singular values $\lambda_1$ significantly more $\lambda_n$, a $\lambda_n$ - a small value close to zero, here$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq 0$ singular values of the matrix $U$. As can be seen from equation (2), small singular values will have a significant impact on the estimated parameter vector. The estimated parameter vector is significantly amplified by small singular values, making the least-squares estimate extremely unreliable and making it impossible to obtain an accurate estimate of the parameters.

To improve the stability of the estimate, Tikhonov's regularization method adds a robust functional constraint to the classical least-squares correction criterion and introduces a regularization factor to regulate the balance, so that an ill-posed problem is transformed into a well-posed problem [14], [15]. The regularization criterion is expressed as:

$$\Phi = (U^T U + \beta\Omega(\hat{\Theta}))U^T\hat{Y} = min,$$

where $\Omega(\hat{\Theta})$ is called a stable functional and plays a stabilizing role; $\beta$ is a regularization parameter that plays the role of two parts in the balance criterion function.

Choice $\Omega(\hat{\Theta})$ often indicates the characteristics of the problem [14]. Different ill-conditioned problems can have different types of stable functionals. We take instead $\Omega(\hat{\Theta})$ - $\hat{\Theta}^T R\hat{\Theta}$ [16], where $R$ is a symmetric non-negative definite matrix, called the regularization matrix. Then the regularization criterion can be expressed as

$$\Phi = U^T U + \beta\hat{\Theta}^T R\hat{\Theta} = min.$$

Various forms $\Omega(\hat{\Theta})$ are reflected by different forms of the regularization matrix $R$. When the parameters contain a priori information, the paper [15], [16] proposes to use the weighted fitting method to determine $R$ based on the a priori information of the parameters. In this case, it is necessary to first perform the singular value decomposition of the matrix of normal equations, and then use the matrix after the singular value decomposition to construct $R$. When the parameters do not contain a priori information, the stability functional is often used as a second norm constraint on the parameters, i.e. $R = I$, and the regularization criterion is:

$$\Phi = U^T U + \beta \hat{\Theta}^T \hat{\Theta} = min.$$

The result of the adjustment is as follows:

$$\Phi = (U^T U + \beta I) U^T \hat{Y} = min.$$

This is a widely used ridge regression method [17]. In this case, the stable functional has the form $\Omega(\hat{\Theta}) = \hat{\Theta}^T R \hat{\Theta} = \hat{\Theta}^T \hat{\Theta}$.

Regularization is a biased estimation method. While improving the ill-conditioned nature of the normal equation, it inevitably introduces bias [13]-[15]. When the ill-conditioned nature of the normal equation is effectively improved, the introduced bias reduces the reliability of the parameter estimates. Currently, research on regularization methods mainly focuses on the selection of the regularization parameter, while there is less research on regularization matrices. A good regularization matrix can effectively improve the ill-conditioned nature of the matrix, minimize the introduction of bias, and make the solution of ill-conditioned problems more reliable. The choice of the regularization matrix is of great importance for solving ill-conditioned problems.

## 2.3 Research of the Method of Constructing Regularized Matrices

Ridge regression estimation can be viewed as a regularization method, where the regularization matrix is the identity matrix. It can effectively reduce the variance of the parameter estimates and improve the stability of the solution. Based on the law of covariance propagation, the formula for calculating the covariance of a ridge regression estimate given in [17], [18]:

$$cov(\hat{\Theta}) = \sigma_0^2 (U^T U + \beta I)^{-1} U^T U (U^T U + \beta I)^{-1}.$$

The trace of the covariance matrix can reflect the total variance of the estimate. The trace can be obtained from the covariance formula.

$$D(\hat{\Theta}) = tr[\sigma_0^2 (U^T U + \beta I)^{-1} U^T U (U^T U + \beta I)^{-1}]. \quad (8)$$

Decomposition of a square matrix $U^T U$ on the eigenvalues gives

$$U^T U = G \Lambda G^T = \sum_{i=1}^{n} G_i \Lambda_i G_i^T. \quad (9)$$

In the formula, the eigenvalue $\Lambda_i = \lambda_i^2$, that is, the eigenvalue of the coefficient matrix of the normal equation is equal to the square of the singular value of the coefficient matrix of the observed equation, and the eigenvector of the eigenvalue $\Lambda_i$ is a right singular vector $\lambda_i$. Substituting equation (9) into equation (8), we obtain:

$$D(\hat{\Theta}) = tr\left[\sigma_0^2 \left( (U^T U)^{-1} \left( \sum_{i=1}^{n} G_i \Lambda_i G_i^T + \right.\right.\right.$$

$$\left. + \beta \sum_{i=1}^{n} G_i G_i^T \right)^{-1} (U^T U + \beta I)^{-1}] =$$

$$= \sigma_0^2 \sum_{i=1}^{n} \frac{\Lambda_i}{(\Lambda_i + \beta)^2}. \quad (10)$$

The ridge estimate is a biased estimate and its bias is calculated using the formula [18]:

$$\left(bias(\hat{\Theta}_\beta)\right)^2 = \beta^2 (U^T U + \beta I)^{-1} \times U U^T (U^T U + \beta I)^{-1}. \quad (11)$$

Equation (11) is simplified by expansion into eigenvalues, and the following is obtained:

$$tr\left[\left(bias(\hat{\Theta}_\beta)\right)^2\right] = \sum_{i=1}^{n} \hat{\Theta}^T G_i \frac{\beta^2}{(\Lambda_i + \beta)^2} G_i^T \hat{\Theta}. \quad (12)$$

Selective regularization of small eigenvalues improves the stability of ill-conditioned problems, reducing the variance of estimates and minimizing the introduced bias during parametric identification.

## 2.4 Construction of a Representative Set of Models

Representative set of models $\Omega_\Theta$ for the identification process satisfying assumptions ($a$)-($e$), is considered as a set of model parameters $\Theta$, defined as follows:

$$P(\Theta \notin \Omega_\Theta) < \alpha, \ 1 > \alpha > 0,$$

where $P$- probability, and $\alpha$ - confidence coefficient. The representative set for models in form (4) is denoted by $\Omega_{\tilde{\Theta}}$, and for models in form (2) - through $\Omega_{\hat{\Theta}}$. Representative set $\Omega_{\tilde{\Theta}}$ can be defined as a parallelepiped:

$$\Omega_{\tilde{\Theta}} = \{\Theta: \Theta = [\vartheta_1, \ldots, \vartheta_m]$$
$$\vartheta_i \in \langle \tilde{a}_i - \mu\delta_i, \tilde{a}_i + \mu\delta_i \rangle\}, i = 1, \ldots, m. \quad (13)$$

The probability that $\Theta \in \Omega_{\tilde{\Theta}}$ determined by virtue of independence $\sigma_i$ (6) as follows:

$$P(\Theta \in \Omega_{\tilde{\Theta}}) = \prod_{i=1}^{m} P(\vartheta_i \in \langle \tilde{a}_i - \mu\delta_i, \tilde{a}_i + \mu\delta_i \rangle).$$

For this value of the confidence coefficient $a$, using normal distribution tables, you can determine the coefficient $\mu$ sets $\Omega_{\tilde{\Theta}}$. The representative set can also be constructed in another form:

$$\Omega_{\tilde{\Theta}}^T = \{\Theta: \Theta = [\vartheta_1, \ldots, \vartheta_m],$$
$$\vartheta_i \in \tilde{a}_i + (\lambda_{2i} - \lambda_{2i})\wp\delta_i, \lambda_j \geq 0, \sum_{j=1}^{2m} \lambda_j \leq 1, \quad (14)$$
$$i = 1, \ldots, m\}.$$

This set is closed in the convex hull $C(\Omega_{\tilde{\Theta}}^T)$ with $2m$ vertices that form a set

$$X_{\tilde{\Theta}} = \{\tilde{\Theta}: \tilde{\Theta} = [\vartheta_1, \ldots, \vartheta_m], \vartheta_j = \tilde{a}_j,$$
$$\vartheta_i = \tilde{a}_i - \wp\delta_i \cup \tilde{a}_i + \wp\delta_i, i = j, i = 1,2, \ldots m\}.$$

It can be shown that for independent distributions $\sigma_i$ set $\Omega_{\tilde{\Theta}}^T$ with $\wp = \mu(m!)^{1/m}$ provides at least the same confidence factor $a$, that is the set $\Omega_{\tilde{\Theta}}^T$.

Transformations (5) of sets $\Omega_{\tilde{\Theta}}^T$ and $X_{\tilde{\Theta}}^T$ let us denote by $\Omega_{\hat{\Theta}}^T$ and $X_{\hat{\Theta}}^T$.

Set of vertices $X_{\hat{\Theta}}^T$ generates a convex hull $C(\Omega_{\hat{\Theta}}^T)$ over a set $\Omega_{\hat{\Theta}}^T$ and can be defined as follows:

$$X_{\hat{\Theta}}^T = \{\hat{\Theta}: \hat{\Theta} = A^{-1}\tilde{\Theta}, \tilde{\Theta} \in X_{\tilde{\Theta}}^T\}.$$

## 2.5 Evaluating the Dynamic Properties of a Process Model Using a Representative Set $\Omega_\Theta$

Representative set $\Omega_\Theta$ can be used to determine the range of change in gain and frequency characteristics when $\omega \in < 0, \omega_s >$, where $\omega_s$- Shannon frequency. Using the following inequality

$$min\{f_{\Theta_1}(\omega), f_{\Theta_2}(\omega)\} \leq f_\Theta(\omega) \leq$$
$$max\{f_{\Theta_1}(\omega), f_{\Theta_2}(\omega)\}, \omega \in \langle 0, \omega_s \rangle \quad (15)$$

it is possible to find extreme values on a set of vertices $X_{\hat{\Theta}}^T$ to calculate the range of variation of the model characteristics due to identification errors [19], [20]. Since it is desirable to have a smaller number peaks in $X_{\hat{\Theta}}^T$, then it is preferable to use a set in the form (14) than in the form (13). Here $f_{\Theta^T}(\omega)$ means a continuous functional of the vector of model parameters $\Theta$ and frequencies $\omega$, $\omega \in \langle 0, \omega_s \rangle$

Let's consider the properties of moving average models:

$$y(kT^*) = \sum_{i=0}^{R} b_i u((k - d - i)T^*), \quad (16)$$

where $y$ - is the output signal, $u$ - is the input signal, $T^*$ - sampling interval, $d$ - discrete value of delay, and parameters $b_i$ form a vector of unknown parameters $\hat{\Theta}$ in model (2).

The gain of the model is $K = \sum_{i=0}^{R} b_i$.

Let us denote by $K_1$ and $K_2$ gain values for parameter vectors $\Theta_1 = [b_{01}, \ldots, b_{R1}]^T$ and $\Theta_2 = [b_{02}, \ldots, b_{R2}]^T$ accordingly, and through $K(\lambda)$ - gain factor for the parameter vector

$$\Theta(\lambda) = \Theta_1\lambda + \Theta_2(1 - \lambda), \lambda \in \langle 0,1 \rangle.$$

Then:

$$K(\lambda) = \sum_{i=0}^{R} [\lambda b_{i1} + (1 - \lambda) b_{i2}] = \lambda K_1 + (1 - \lambda) K_2.$$

Hence $min(K_1, K_2) \leq K(\lambda) \leq max(K_1, K_2)$, i.e., a relation equivalent to (15) is obtained when as a functional $f_\Theta(\omega)$ the gain factor $K$ is taken. This ratio determines the extreme values of the static gain factor.

Consideration of the frequency characteristics of model (16), obtained by substitution $z = exp(-j\omega T^*)$ into the discrete transfer function (16), allows us to obtain a relation that is equivalent to relation (15) with $f_\Theta(\omega) = \phi_\Theta(\omega)$, $\phi_\Theta(\omega)$- phase-frequency characteristic. This ratio determines the extreme values of the phase-frequency characteristic, especially for frequencies corresponding to the point (-1,0) on the complex plane.

It is also possible to determine the upper limit of the amplitude-frequency characteristic and obtain an estimate for the lower limit of the amplitude-frequency characteristic (Fig. 1).

# 3 RESULTS AND DISCUSSION

Numerical example. To demonstrate the proposed procedure, a third-order moving-average model was considered in a passive experiment with $M = 200$ samples and sampling interval $T_s = 1.0$. The nominal parameter vector was chosen as $\Theta_0 = [0.80, 0.35, 0.15]^T$, and an unmeasured disturbance $\varepsilon(k)$ was added to the output so that the matrix $U^T U$ became poorly conditioned.

For robust estimation, Tikhonov regularization with R = I was applied. The regularization parameter was selected by the L-curve principle; for the case shown in Figures 1 and 2, $\beta = 0.10$ and the representative-set confidence coefficient was $\alpha = 0.95$. The noise level $\sigma_n$ was varied in the interval 0-0.20.

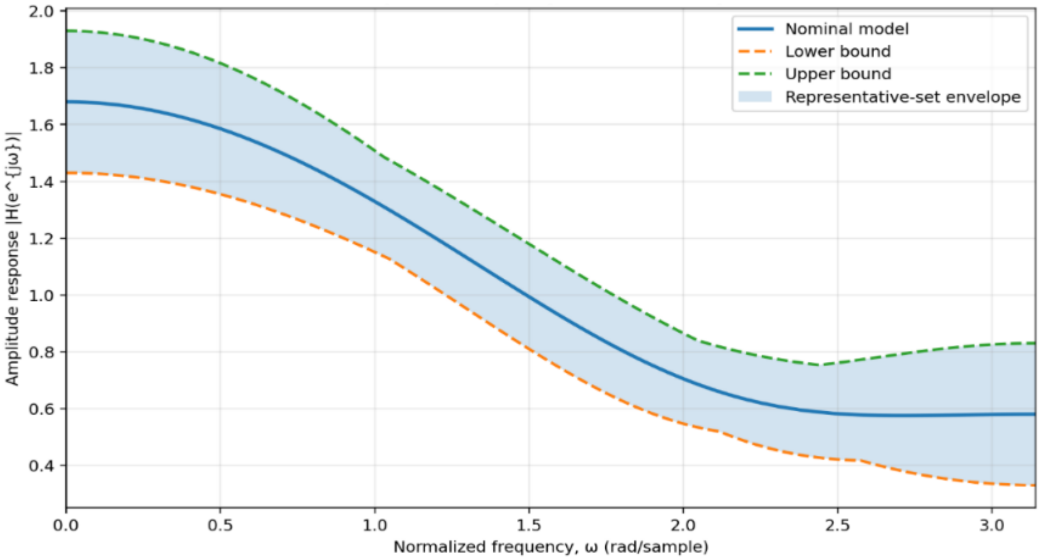


Figure 1: Amplitude-frequency response of the nominal process model and the upper/lower bounds obtained from the representative set ($M = 200$, $\alpha = 0.95$).

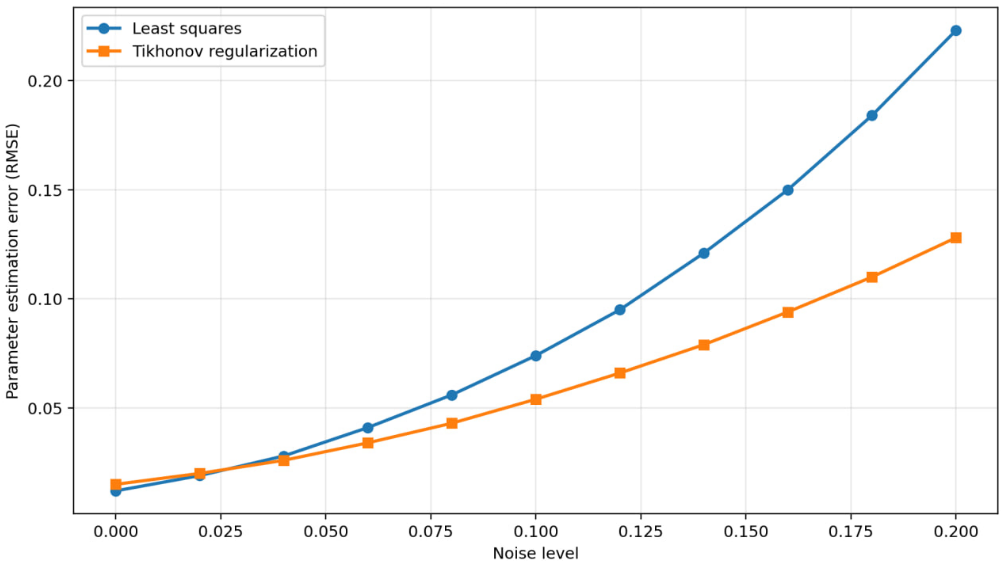


Figure 2: Comparison of parameter estimation error for the classical least squares method and the Tikhonov-regularized approach under increasing noise level (R = I, $\beta = 0.10$).

Figure 1 presents the nominal amplitude-frequency response together with the upper and lower bounds obtained from the representative set. Figure 2 shows that the parameter-estimation error of the classical least-squares method increases faster with the noise level, whereas the regularized estimate remains more stable.

From a practical point of view, the proposed method can be used in software-based identification and monitoring modules of multilevel control systems, where measurement noise and unmeasured disturbances reduce the reliability of classical least-squares estimates. The obtained bounds for amplitude- and phase-frequency characteristics can support robust controller tuning and engineering decisions under uncertainty.

# 4 CONCLUSIONS

In conclusion, let us dwell on the properties of the process described by the autoregressive moving average model:

$$\sum_{i=0}^{R} a_i y\big((k-i)T^*\big) = \sum_{i=1}^{R} b_i u\big((k-d-i)T^*\big),\ a_0 = 1, \quad (17)$$

where are the coefficients $a_i$ and $b_i$ form a vector of model parameters $\Theta = [a_1, a_2, \dots, a_R, b_1, \dots, b_R]^T$. In (17), the indexing is intentional: the autoregressive part starts from $i$ = 1, whereas the moving-average/noise part includes the current sample $i$ = 0. For the static gain $K$ of model (13), the relation can be obtained

$$K = \sum_{i=1}^{R} b_i / \sum_{i=0}^{R} a_i,$$

equivalent to (15), which determines the extreme values of $K$.

Using substitution $z = exp(-j\omega T^*)$ in the discrete transfer function of model (17) allows us to obtain a relation that is equivalent to (15) with $f_\Theta(\omega) = \phi_\Theta(\omega)$. This relationship determines the extreme values of the phase-frequency characteristic for frequencies corresponding to the point (-1, 0) on the complex plane, for sufficiently small changes in the vector of model parameters.

Finally, we can estimate the lower and upper limits for the amplitude-frequency characteristic of the model (17).

Figures 1 and 2 confirm the comparative robustness of the proposed identification procedure on the numerical example. As the noise level increases, the parameter-estimation error of the classical least-squares method grows more rapidly, whereas the Tikhonov-regularized solution remains more stable. Therefore, regularization provides tighter and more reliable bounds of the dynamic characteristics under uncertainty.

# REFERENCES


[1] V. M. Ponyatsky and T. Zung, "Estimation of the coefficients of a dynamic object model using the finite-frequency identification method under passive experimental conditions," Bulletin of Tula State University. Technical Sciences, no. 2, pp. 247-254, 2013.

[2] S. Li, J. Yang, M. Iwasaki, and W. H. Chen, "Hierarchical disturbance/uncertainty estimation and attenuation for integrated modeling and motion control: Overview and perspectives," IEEE/ASME Trans. Mechatronics, vol. 30, no. 6, pp. 4435-4449, Dec. 2025, [Online]. Available: https://doi.org/10.1109/TMECH.2024.3515084.

[3] M. Yu. Ryabchikov and E. S. Ryabchikova, "Identification of an object model in the presence of unknown disturbances with a wide frequency range based on the transition to signal increments and data sampling," Computer Research and Modeling, vol. 16, no. 2, pp. 315-337, 2024.

[4] X.-J. Wei, Z.-J. Wu, and H. R. Karimi, "Disturbance observer-based disturbance attenuation control for a class of stochastic systems," Automatica, vol. 63, pp. 21-25, 2016, [Online]. Available: https://doi.org/10.1016/j.automatica.2015.10.019.

[5] I. I. Jumanov, S. M. Xolmonov, and O. I. Djumanov, "Identification based on the optimal information structure of non-stationary objects," in Proc. 2025 Int. Russian Smart Industry Conf. (SmartIndustryCon), 2025, [Online]. Available: https://doi.org/10.1109/SmartIndustryCon65166.2025.10986122.

[6] G. Chen, C. Mehdi-Souzani, and N. Anwer, "Error source identification in metrology digital twin systems using machine learning," Advanced Engineering Informatics, vol. 70, pp. 1-11, 2026.

[7] N. R. Yusupbekov, H. Z. Igamberdiev, and U. F. Mamirov, "Algorithms of sustainable estimation of unknown input signals in control systems," J. Autom., Mobile Robot. Intell. Syst., vol. 12, no. 4, pp. 83-86, 2018, [Online]. Available: https://doi.org/10.14313/JAMRIS4-2018/29.

[8] I. Jumanov and S. Kholmonov, "Identification of non-stationary objects based on training a neural network and tuning the parameters of a generalized model," in Proc. 4th Int. Conf. Energetics, Civil and Agricultural Engineering, Tashkent, Uzbekistan, 2023, E3S Web Conf., vol. 434, Art. no. 02002, [Online]. Available: https://doi.org/10.1051/e3sconf/202343402002.

[9] P. V. de C. Souza, A. J. Guimarães, T. S. Rezende, V. S. Araujo, L. F. do Nascimento, and L. Oliveira Batista, "An intelligent hybrid model for the construction of expert systems in malware detection," in Proc. 2020 IEEE Conf. Evolving and Adaptive Intelligent Systems (EAIS), Bari, Italy, 2020, pp. 1-8, [Online]. Available: https://doi.org/10.1109/EAIS48028.2020.9122770.

[10] C. L. Giles, R. Sun, and J. M. Zurada, "Neural networks and hybrid intelligent models: Foundations, theory, and applications," IEEE Trans. Neural Netw., vol. 9, no. 5, pp. 721-723, Sep. 1998, [Online]. Available: https://doi.org/10.1109/TNN.1998.712147.

[11] C. L. Lawson and R. J. Henson, Solving Least Squares Problems. Englewood Cliffs, NJ, USA: Prentice-Hall, 1986.

[12] H. Z. Igamberdiev and U. F. Mamirov, "Regular algorithms for the parametric estimation of the uncertain object control," in Proc. 11th World Conf. "System for Industrial Automation" (WCIS), in Advances in Intelligent Systems and Computing, vol. 1323, pp. 322-328. Cham, Switzerland: Springer, 2021, [Online]. Available: https://doi.org/10.1007/978-3-030-68004-6_42.

[13] A. A. Bobachev, A. V. Deshcherevskii, and A. Y. Sidorin, "Regularization algorithms for increasing the stability of the solution to the inverse problem," Seism. Instrum., vol. 57, pp. 409-423, 2021, [Online]. Available: https://doi.org/10.3103/S0747923921040022.

[14] A. N. Tikhonov and V. Ya. Arsenin, Methods for Solving Ill-Posed Problems. Moscow, Russia: Nauka, 1986. (In Russian).

[15] R. C. Aster, B. Borchers, and C. H. Thurber, "Tikhonov regularization," in International Geophysics, vol. 90, pp. 89-118, 2005, [Online]. Available: https://doi.org/10.1016/S0074-6142(05)80019-1.

[16] J. Zhu, Y. Tian, and X. Tao, "United expression and solution of adjustment criteria with parameters," Acta Geodaetica et Cartographica Sinica, vol. 41, no. 1, pp. 8-13, 2012.

[17] M. F. Yassen, F. S. Al-Duais, and M. M. A. Almazah, "Ridge regression method and Bayesian estimators under composite LINEX loss function to estimate the shape parameter in Lomax distribution," Comput. Intell. Neurosci., vol. 29, Aug. 2022, [Online]. Available: https://doi.org/10.1155/2022/1200611.

[18] A. Irandoukht, "Optimum ridge regression parameter using R-squared of prediction as a criterion for regression analysis," J. Stat. Theory Appl., vol. 20, no. 2, pp. 242-250, Jun. 2021, [Online]. Available: https://doi.org/10.2991/jsta.d.210322.001.

[19] J. Bai, "Estimation of a change point in multiple regression models," Rev. Econ. Stat., vol. 79, no. 4, pp. 551-563, 1997.

[20] J. U. Sevinov, O. K. Boborayimov, and N. H. Bobomurodov, "Algorithms for synthesis of adaptive neural network control systems based on the velocity gradient method," in Proc. 16th Int. Conf. Applications of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools (ICAFS), in Lecture Notes in Networks and Systems, vol. 1141. Cham, Switzerland: Springer, 2023, [Online]. Available: https://doi.org/10.1007/978-3-031-76283-3_34.