# Symbolic Stability Analysis of Impulse Energy Converters in Parallel Operation Mode Using a Digital Linear Model

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Abstract: A methodology for applying computer programs to symbolic stability analysis of parallel-operating electric energy pulse converters is proposed. A method is proposed for interaction between the programs FASTMEAN and Maple aimed at finding an analytical expression for the equivalent operator transfer function that describes stability conditions of a complex dynamic system. The simulation program FASTMEAN used in Russian universities of telecommunications has a block for symbolic analysis capable of generating analytical expressions for the Laplace transform determinant of a circuit, currents, and voltages in complex electronic circuits. A linear computational model of parallel connection of PWM converters suitable for stability analysis was developed. Using this program, analytical expressions were obtained for the main determinant of a linear circuit containing N identical converter modules with negative feedback. It is suggested that these expressions be transferred into the symbolic mathematics software Maple for their structural transformation and mathematical processing. As a result of formula processing by Maple, an analytical expression of equivalent operator transfers function B(p) along the feedback loop describing multi-channel system stability is generated. For an arbitrary number of parallel-connected pulse converters, a general analytical expression for B(p) is found from which frequency dependence graphs of magnitude and phase are constructed. Based on these dependencies, stability margins of the pulsed system are evaluated. To verify the reliability of the proposed method, time-domain transient simulation modelling is performed. The proposed method of symbolic stability analysis for linear models describing the parallel operation mode of pulse con verters not only predicts instability conditions for the entire system but also opens up the possibility of synthesizing optimal feedback correction circuits that provide deep feedback with a maximum number of identical modules.

## 1 INTRODUCTION

Nowadays the software systems of symbolic mathematics or computer algebra have become an indicator of the power of computers. In accordance with this requirement all computer mathematical systems were equipped with symbolic modules. Some companies developed own product (Maple, Mathematica), others purchased and adopted it to their interface (MATLAB, Mathcad). However, conventional programs for electrical circuit analysis that are widely used in education (Pspice, Micro-Cap, Win Spice, ets.) solve the tasks only numerically.

FASTMEAN 6.1 simulation program [1] is based on new algorithms for fast numerical solution of matrix differential state equation of the circuit. In addition, the program is equipped with a symbolic analysis module based on the innovative method of circuit determinants [2], The fast numerical solvers referenced here underlie FASTMEAN's conventional time-domain and frequency-domain circuit simulation capabilities (used, for example, to numerically cross-check the transient response in Section 3), whereas the present study relies specifically and exclusively on the separate symbolic analysis module described next, which analytically derives closed-form transfer functions rather than numerically solving the state equations; the two capabilities are complementary features of the same software package, and only the latter is central to the methodology of this paper. [3]. This method is designed to obtain the optimal complexity of symbolic expressions for transfer functions of electrical circuit, bypassing the construction of auxiliary mathematical models in the form of equations, matrices and graphs. However, for complex circuits it is not possible to obtain an analytical expression in the form of a fractional rational function. For this reason, the resulting expression from FASTMEAN as a text file is transferred to Maple for further processing and analysis. It is also possible to return the resulting expression from Maple to FASTMEAN in order to compare analytical and numerical solutions. A large number of scientific and technical publications are devoted to the study of theory and methods for designing highly efficient electrical energy converters with pulse-width modulation (PWM) covered by negative feedback (NFB). The fundamental principles of switching-mode power supply design and feedback organization are also widely described in the specialized power electronics literature [4]. Currently, such devices are created in the form of functionally complete transistor modules with digital control, which find wide application in power electronics. Each module performs the conversion of an unsterilized constant voltage from a primary source (for example, a battery) into a specified stabilized output voltage. The stabilization mechanism is based on negative feedback principles, i.e., each module contains a power stage and a low-power control loop that compensates for deviations of the output voltage from the given reference value. To ensure high stability of the output voltage under varying load conditions and unstable primary source voltage, it is necessary to apply sufficiently deep negative feedback with predetermined margins of stability. Simple linear models are known that allow determining the transfer function of a feedback loop in such a device and assessing its stability based on well-known frequency criteria such as Nyquist or Bode [2], [3]. Phase-based and optimization-oriented methods have also been applied to the stability analysis of digital dynamic systems [5]. Recent studies have further extended frequency-domain stability analysis to advanced power electronic converters and conditionally stable electronic circuits [6], [7]. These models accurately describe the dynamic properties of the converter for harmonic disturbances up to a frequency equal to half the clock frequency of the pulse control block.

The most effective for practical use is the automated closed-loop method [8]-[11], which allows not only to calculate the loop gain function of a linear model but also to measure frequency characteristics in a real device. However, the application of such an efficient method is limited only to one autonomously operating module of a pulse converter. This method cannot be extended to more complex power module architectures, in particular their parallel connection. At the same time, the task of increasing the output power of DC-DC conversion based on parallel connection of N standard (identical) modules remains relevant and attracts the attention of power electronics developers. The main task arising in the study and design of parallel operation mode is to ensure stability when all N modules interact with each other.

The objective of this work is to substantiate a computer model of the linear equivalent circuit for parallel connection of pulse converters with feedback control, demonstrate the methodology of symbolic stability analysis by means of computer simulation, evaluate the influence of the number of modules on the stability margin and carry out an imitation modeling of processes in order to verify the obtained results.

The intended contribution of the authors is: a) to demonstrate a methodology for interaction between the programs FASTMEAN and Maple for symbolic stability analysis; b) to open the possibility of automatic generation and mathematical processing of analytical expressions for equivalent operator transfer functions of complex pulse systems.

# 2 LINEAR COMPUTER MODEL AND SYMBOLIC STABILITY ANALYSIS

## 2.1 Justification of the Structure of a Linear Model for Parallel Connection of PWM Converters

For parallel-connected modules (Fig. 1), the closed-loop method does not work correctly because the characteristics of the loop gain in which a disturbance is introduced will be affected by the remaining N-1 modules and the feedback loop gain will be equal to 0 dB at low frequencies.

Thus, to assess the stability and determine the reserves for gain and phase enhancement of a parallel connection of N converter modules, it is necessary to take into account the influence of the modules on each other. In this case, the Nyquist stability criterion must correctly describe a system with multi-channel negative feedback. A fundamental work on the theory of negative feedback [2] provides a universal formula for the return difference of feedback of a linear electrical circuit

$$F(p) = \Delta(p)/\Delta^{(0)}(p), \tag{1}$$

where $\Delta(p)$ is the main circuit determinant of a network with multi-channel (multi-loop) negative feedback, $\Delta^{(0)}(p)$ is the determinant of the circuit obtained from the main one under the condition that all amplifying elements are switched off for energy direct transmission paths from source to load.

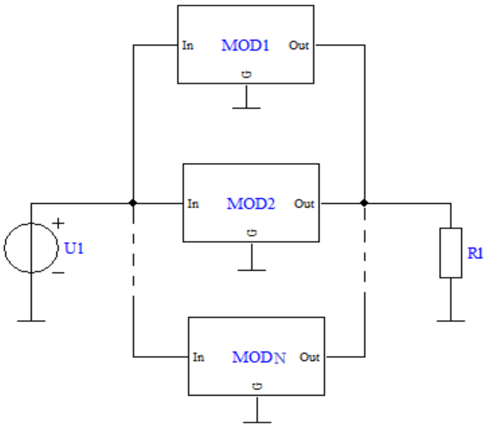



Figure 1: Parallel connection of impulse modules.

Considering that the equivalent loop gain Function $B(p)$ of a system with multi-channel feedback and the return difference defined by formula (2.1) are related by a simple relationship

$$B(p) = 1 - F(p), \tag{2}$$

in fact, formula (2.1) can be used to estimate stability.

Thus, the computer analysis of stability for parallel connection of power converter modules implies the necessity to form by a program the main circuit determinant $\Delta(p)$ in analytical and then numerical forms. This option of symbolic analysis is available in the electronic circuit's simulation software FASTMEAN 6.1 [1].

As an illustration of the above statements, let us consider a simple linear model of a pulse converter with negative feedback shown in Figure 2.

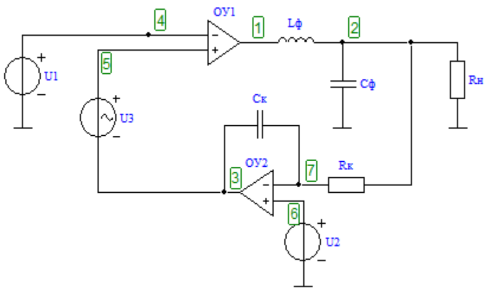
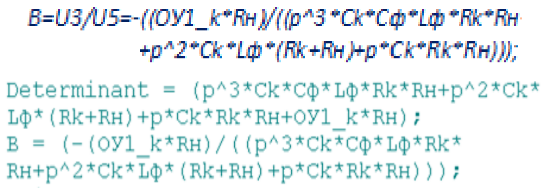



Figure 2: Linear model of one converter module.

In this model, the unstable input voltage *U1* is compared with a feedback signal, amplified by operational amplifier ОУ1 and converted into stabilized load voltage on resistor *Rн*, set by reference voltage *U2=Uref*. The smoothing filter *LфCф* suppresses high-frequency ripples. The correcting circuit *RkCk* of negative feedback loop is implemented as an integrator based on ideal operational amplifier ОУ2.

For this circuit, we will define the loop gain transfer function $B(p)$ in two ways: firstly by using a closed-loop method introducing harmonic perturbation *U3* and calculating the ratio of node voltages on the ends of the disturbing voltage source; secondly by computing the main circuit determinant $\Delta(p)$ and the determinant $\Delta^{(0)}(p)$. Figure 3 presents analytical expressions for the loop gain $B(p)$, formulated for this scheme with both methods utilizing the symbolic analysis block of program FASTMEAN 6.1. Comparison shows that these expressions are fully identical.



```
B=U3/U5=-((ОУ1_k*Rн)/((p^3*Ck*Cф*Lф*Rk*Rн
        +p^2*Ck*Lф*(Rk+Rн)+p*Ck*Rk*Rн)));
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```
Determinant = (p^3*Ck*Cф*Lф*Rk*Rн+p^2*Ck*
Lф*(Rk+Rн)+p*Ck*Rk*Rн+ОУ1_k*Rн);
B = (-(ОУ1_k*Rн)/((p^3*Ck*Cф*Lф*Rk*
Rн+p^2*Ck*Lф*(Rk+Rн)+p*Ck*Rk*Rн)));
```

Figure 3: Screenshots of analytical expressions for the loop gain function *B(p)* and circuit determinant generated by the FASTMEAN 6.1.

Thus, to find the equivalent loop gain function of an arbitrary number of parallel-connected modules, one can use formula (2.1), where determinants Δ(p) and $\Delta^{(0)}(p)$ are automatically calculated using a procedure for computer symbolic analysis.

The proposed methodology also opens up the possibility of synthesizing optimal correcting circuits for transistor modules based on analytical solutions. Further mathematical processing of a complex formula for an operator transfer function formed by the FASTMEAN 6.1 program can be carried out through its interaction with the well-known computer algebra system Maple.

## 2.2 Symbolic Stability Analysis of Parallel-Connected Modules

For the analysis of stability of two or more parallel-connected modules, we use expression (2) applied to the function of equivalent loop B(p). The computer model scheme for two parallel-connected modules is shown in Figure 4.

Using the symbolic analysis block of the FASTMEAN 6.1 program, we will form an analytical expression for the main schematic determinant Δ(p). The found expression is shown in Figure 5. FASTMEAN's symbolic analysis block constructs Δ(p) algorithmically from the circuit topology using a symbolic modified nodal analysis (MNA) procedure: it builds the circuit's admittance/impedance matrix with each element (resistor, inductor, capacitor, controlled source) entered as a symbolic Laplace-domain parameter rather than a numeric value, and then evaluates the determinant of this symbolic matrix (following the classical topological-determinant approach described by Bode) using computer-algebra polynomial expansion, yielding Δ(p) as a closed-form polynomial in the Laplace variable p and the circuit's symbolic component parameters. Also given there Δ0(p) is a formula for the determinant obtained from the main schematic determinant by zeroing out the gain coefficients of power amplifiers in direct energy transmission paths. Using these formulas, it is possible to determine the return difference F(p)\(2.1) and the equivalent loop gain function (B\left(p\right)(2.2). An analytical expression for the loop gain function is also shown in Figure 5.

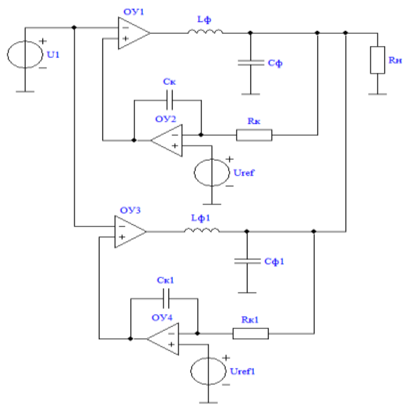


Figure 4: Linear model of two parallel-connected modules of pulse converters.

```
Determinant=2*Cк*p^2*Lф*((p^2*Cф*Lф*Rк*Rн+
(1/2)*Lф*(Rк+2*Rн)*p+Rн*Rк)*p*Cк+K*Rн)*Rк
```

$$B := \frac{K\,R_н}{p\,C_к\left(p^2\,C_ф\,L_ф\,R_к\,R_н+\frac{1}{2}\,L_ф\,(R_к+2\,R_н)\,p+R_н\,R_к\right)}$$

Figure 5: Screenshots of analytical expressions for the determinant and loop gain *B(p)* of two parallel connected modules.

For readability, a simplified plain-text rendering of the key expressions shown as screenshots in Figures 5-7 is given here: the main schematic determinant has the general polynomial form Delta(p) = a3*p^3 + a2*p^2 + a1*p + a0, where the coefficients a0-a3 are products and sums of the circuit's symbolic R, L, and C parameters generated by FASTMEAN; the return difference is F(p) = Delta(p) / Delta0(p); and the loop gain function follows as B(p) = F(p) - 1 = [Delta(p) - Delta0(p)] / Delta0(p). The full, unabbreviated symbolic expressions (with all circuit-parameter coefficients spelled out) remain as shown in Figures 5-7 for completeness and traceability to the FASTMEAN/Maple output.

Performing computer symbolic analysis in a similar way as shown above for different values of parallel operating modules, we can analyze the structure of generated analytical expressions for equivalent transfer operator functions according to the NFB loop. For example, for the number of modules *N*=3, corresponding expressions as screenshots are given in Figure 6.

```
Determinant=3*Rк^2*Cк^2*Lф^2*p^4
*(p*(p^2*Cф*Lф*Rк*Rн
+(1/3)*Lф*(Rк+3*Rн)
*p+Rк*Rн)*Cк+K*Rн);
```

$$B := \frac{K\,R_н}{p\,C_к\left(p^2\,C_ф\,L_ф\,R_к\,R_н+\frac{1}{3}\,L_ф\,(R_к+3\,R_н)\,p+R_н\,R_к\right)}$$

Figure 6: Screenshots of analytical expressions for the determinant and loop gain *B(p)* of three parallel connected modules.

```
Determinant= 6*Rк^5*p^10*Cк^5*Lф(p*(p^2*Cф
*Lф*Rк*Rн+(1/6)*Lф*(Rк+6*Rн)*p+Rн*Rк)
*Cк+K*Rн)^5;
```

$$B := \frac{K\,R_н}{p\,C_к\left(p^2\,C_ф\,L_ф\,R_к\,R_н+\frac{1}{6}\,L_ф\,(R_к+6\,R_н)\,p+R_н\,R_к\right)}$$

Figure 7: Screenshots of analytical expressions for the determinant and loop gain *B(p)* of six parallel connected modules.

For six identical parallel operating modules (N=6) with the same element parameters, analytical expressions formed by the programs FASTMEAN 6.1 and Maple 17 are shown in Figure 7.

Analysis of the obtained expressions for the function B(p) for different numbers N of identical parallel operating modules shows the possibility of describing them by a single general formula: In outline, the generalization proceeds as follows: for each specific case (N = 2, 3, 6), the Rк (equivalent single-module feedback resistance) and RH (load resistance) terms in the closed-loop denominator obtained from FASTMEAN/Maple appear in the combination (Rк/N + RH), reflecting the parallel combination of N equal feedback paths acting on the shared load; substituting N = 2, N = 3, and N = 6 into this pattern reproduces the individually derived expressions for those cases, and collecting terms confirms that replacing the case-specific numeric coefficients with the general factor 1/N in front of Rк, while leaving RH unscaled (since the load itself does not multiply with N), yields Equation (3) as the common closed form valid for arbitrary N.

$$B(p) = \frac{KR_{\text{н}}}{pC_{\text{к}}\left(p^2 C_{\text{ф}} L_{\text{ф}} R_{\text{к}} R_{\text{н}} + \frac{1}{N} L_{\text{ф}} (R_{\text{к}} + NR_{\text{н}}) p + R_{\text{к}} R_{\text{н}}\right)}. \quad (3)$$

Thus, the proposed methodology allows finding in analytical form an equivalent operator transfer function B(p) of loop gain for linear models of electronic devices with multi-loop feedbacks. This function, when assigned numerical values of parameters, reliably describes system stability in the frequency domain, enables estimation of stability margins and achievable depth of negative feedback. As the number of modules increases, the functions B(p) generated by the programs FASTMEAN 6.1 and Maple 17 also makes it possible to assess changes in amplitude and phase stability margins.

## 2.3 Effect of the Number of Modules on Gain and Phase Stability Margins

Figure 8 shows the frequency characteristics of the loop gain function B(j\omega) for an equivalent linear circuit with numerical values of element parameters in the main system of units: Koy1=0.5; Lф=50uH; Cф=10uF; Ck=10nF; Rk=10kOm; Rн=5Om.

Comparison of these characteristics (Fig. 8) with similar ones for two or more parallel-connected modules reveals that their asymptotic behavior at infinitely low and high frequencies is identical. The difference in frequency characteristics occurs at medium frequencies, leading to changes in stability margins of systems composed of parallel-connected identical pulse converters as their number increases. Table 1 presents the dependencies of gain and phase stability margins for different numbers of parallel-connected modules. The gain and phase margins in Table 1 were determined directly from the open-loop Bode magnitude and phase plots generated by Maple from the symbolic transfer function B(p): the gain margin is the negative of the open-loop gain (in dB) at the frequency where the phase crosses -180 degrees (the phase-crossover frequency), and the phase margin is 180 degrees plus the open-loop phase (in degrees) at the frequency where the gain crosses 0 dB (the gain-crossover frequency), following the standard Bode stability-margin definitions. For brevity, only the Bode plots for the representative cases N = 2, 3, and 6 are shown in Fig. 8; the corresponding plots for N = 8 and N = 10 were generated using the same procedure and are omitted from the figure for space, but the resulting margin values are reported in Table 1.

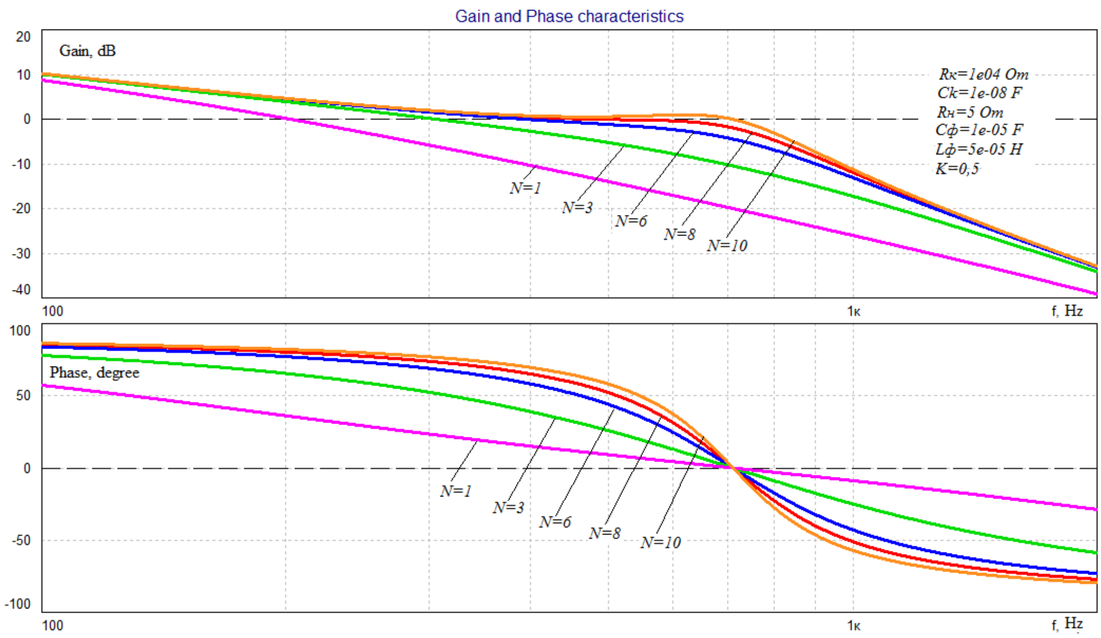


Figure 8: Gain and phase-frequency characteristic of the transfer function $B(p)$ for different numbers of modules.

Table 1: Dependencies of gain and phase stability margins on the number of parallel-connected modules.

| Number of modules, N | Gain margin, ΔA dB | Phase margin, Δψ deg |
|---|---|---|
| 1 | 20 | 65.3 |
| 3 | 10.5 | 51.7 |
| 6 | 4.45 | 59.1 |
| 8 | 2.0 | 55.5 |
| 10 | 0.038 | 0.96 |
| N>10 | instability | instability |

As can be seen from this table, with the number of pulse converters modules $N$=10, the stability margins decrease sharply and in fact the system is at the boundary of stability. Further increase in the number of modules is unacceptable due to instability. It should be clarified that "instability" here refers to the practical erosion of the stability margins toward zero (as already evident at N = 10, where the phase margin has fallen to 0.96 degrees) rather than to divergence of B(p) itself: as N tends to infinity, Equation (3) does converge to a finite limiting transfer function, p^2*C_phi*L_phi*R_k*R_H + L_phi*R_H*p + R_k*R_H, so the linear closed-loop system does not become unbounded in the mathematical sense. However, a phase margin approaching zero means the system has essentially no robustness to unmodeled delays, component tolerances, or nonlinearities present in a real converter (switching dead-time, parasitic elements, digital control latency), so that in practice even small deviations from the idealized linear model are expected to drive the real system into sustained oscillation or instability, even though the idealized linear model formally remains marginally stable. This distinction between the idealized linear limit and practical robustness is the basis for characterizing N > 10 as unacceptable.

Therefore, an important practical task is to determine the optimal structure and parameters of the negative feedback loop for a single module of a pulse converter intended to create a stable parallel operation mode with a significant number of modules (for example, much more than ten).

In the next subsection, transient characteristics corresponding to different numbers of identical modules connected in parallel and, accordingly, to various values of stability margins will be considered.

## 2.4 Analysis of Stable Transient Processes and Generation Mode of Parallel Connected Modules

In order to verify the reliability of the proposed method for analyzing stability, we will simulate transient characteristics for different numbers of parallel-connected modules. Figure 9 shows the transient response curves of stable module connections for N = 1, 3, 6.

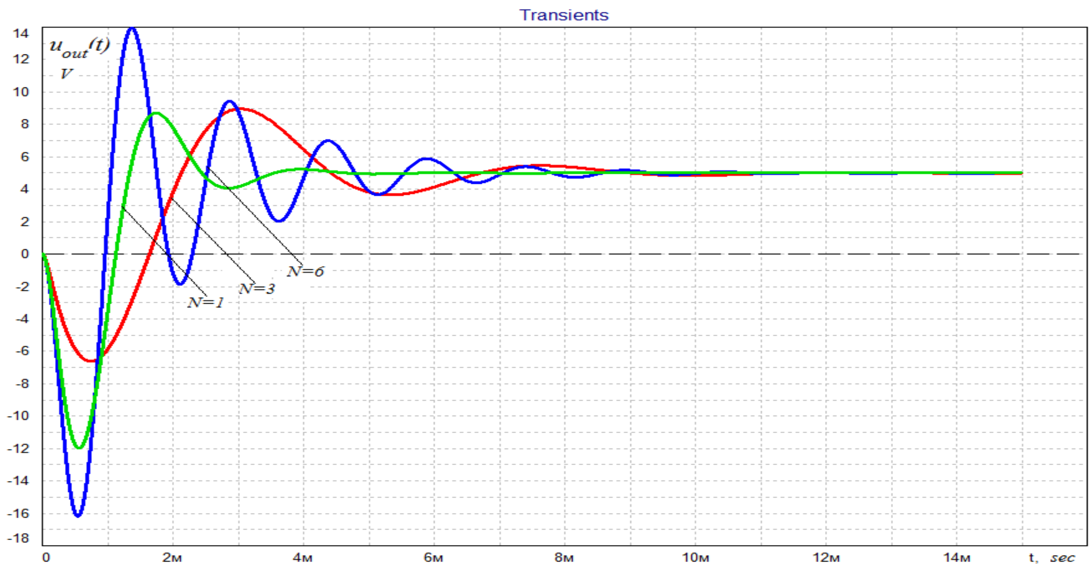


Figure 9: Stable transient characteristic plots for different numbers of modules N=1,3,6.

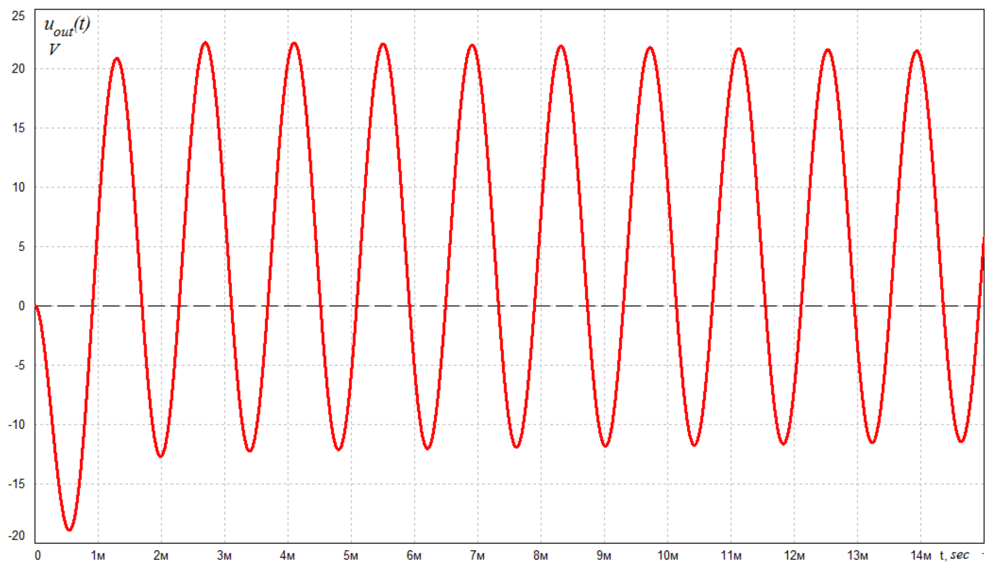


Figure 10: The process at the boundary of stability (N=10).

The transient process at the stability boundary with an equal number of modules $N$=10 is shown in Figure 10. As can be seen from this plot, an undamped oscillatory process arises in the system with a frequency of f = 770 Hz, which corresponds to unacceptably small margins of stability (Table 1). The transient response in Figure 10 was obtained by applying a small step perturbation (a 1% step change in the reference/output current setpoint) to the closed-loop linear model at t = 0, starting from steady-state (zero-error) initial conditions, and simulating the resulting time-domain response of the N = 10 system in FASTMEAN; this step-response test is the standard way of exciting and observing a marginally stable linear system's natural oscillatory mode. As a cross-check, the oscillation frequency observed in Figure 10 (770 Hz) is consistent with the frequency at which the corresponding open-loop Bode phase plot for N = 10 crosses -180 degrees (the phase-crossover frequency used to compute the gain margin in Table 1), confirming that the system in Figure 10 is indeed operating at its predicted stability limit.

# 3 CONCLUSIONS

The proposed method of symbolic stability analysis for linear models describing the parallel operation mode of pulse converters not only predicts instability but also synthesizes optimal correction circuits that provide deep feedback with a maximum number of identical modules. Symbolic methods for investigating complex linear electrical circuits offer advantages over purely numerical methods for this class of problem. Rather than readability alone, since analytical expressions for complex systems can themselves become cumbersome (as the screenshots in Figures 5-7 illustrate), the principal advantage lies in their ability to reveal explicit parameter dependencies, enabling analytical sensitivity and stability analysis (e.g., directly observing how each circuit parameter and the module count N enter the stability margins in Equation (3)) that a purely numerical simulation, which only yields point values for specific parameter sets, does not directly provide. Some problems in analyzing and synthesizing electric circuits can be solved exclusively by symbolic methods. To fully achieve these benefits, efficient programs for symbolic analysis of electric circuits as well as their interaction with computer algebra systems for mathematical transformations and simplification of obtained expressions are required. Structural transformations and mathematical processing performed in Maple yield more meaningful analytical results.

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