Improving the Puma Optimization Algorithm Using a Hybridization of the Gradient-Dependent Religion Algorithm with a New Beta Coefficient

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Abstract: Metaheuristic optimization algorithms often face difficulties in achieving an effective balance between exploration and exploitation, leading to slow convergence or entrapment in local optima. The Puma Optimization (PO) algorithm is a modern metaheuristic known for its strong exploratory capabilities; however, it suffers from deficiencies in refining solutions during the final stages of the optimization process. In this paper, a hybrid model is proposed that combines the PO algorithm with the hybrid Divine Religion Algorithm-Conjugate Gradient (DRA-CG) method, aiming to enhance overall performance. Specifically, PO is utilized for global exploration, while DRA-CG is employed to bolster local exploitation. Furthermore, a novel parameter, β, has been developed within the conjugate gradient component to improve convergence speed and numerical stability. Experimental results demonstrate that the proposed method achieves superior performance in terms of both accuracy and convergence speed, thereby validating its effectiveness in addressing complex optimization problems across diverse benchmark functions and real-world scenarios.

1 INTRODUCTION

Metaheuristic optimization algorithms are considered effective tools for solving complex and nonlinear optimization problems, having demonstrated their capability to handle large, high-dimensional search spaces across various engineering and scientific applications [1], [2]. This field has witnessed remarkable progress in recent years, with numerous modern algorithms being proposed to enhance performance and address issues such as slow convergence or entrapment in local optima [3], [4]. The efficiency of metaheuristic optimization algorithms fundamentally relies on achieving an appropriate balance between the processes of exploration and exploitation; an imbalance in this regard leads to suboptimal performance or premature convergence toward non-ideal solutions. Despite the successes achieved by these algorithms, they continue to face challenges when applied to real-world problems, particularly concerning convergence speed and stability.

The PO algorithm is a modern nature-inspired algorithm that has demonstrated high efficiency in global exploration; however, it suffers from weaknesses in its local exploitation mechanisms, which adversely affects its performance during the final stages of the optimization process. Conversely, the Divine Religion Algorithm (DRA) is a modern algorithm developed to address continuous optimization problems, offering effective search strategies that can be leveraged to enhance performance [5]. In contrast, the conjugate gradient algorithm is regarded as an effective method for local optimization; its performance depends significantly on the selection of the parameter β, which governs the search direction and convergence speed. Numerous modern variants of this algorithm have been developed to improve its performance and enhance its numerical stability [6], building upon established theoretical foundations that guarantee global convergence properties [7]. Accordingly, this research aims to develop a hybrid model that combines the PO algorithm with the DRA-CG algorithm, while proposing a novel parameter-β-to enhance overall performance. The proposed model seeks to achieve a better balance between exploration and exploitation, thereby improving solution accuracy and accelerating the convergence process.

2 PUMA OPTIMIZATION ALGORITHM

The PO algorithm is based on mimicking the animal's natural behavior during hunting, locomotion, and reproduction. The optimal solution within the community is represented by a male puma, while the remaining solutions are represented by female puma. The algorithm is used in two main phases: exploration and exploitation [8].

2.1 Puma Intelligence and Phase Change Mechanism

- First: The Unexperienced Puma Phase. This phase is applied during the first three iterations of the operation. In this phase, exploitation and exploration are performed in parallel. Then performance of each phase is evaluated using two functions:

The first function, f1. This function represents the improvement in the objective function resulting from both the exploration and exploitation phases.

$$f_1^{explore} = PF_1 \times \frac{SeqCost_{Explore}}{SeqTime} \quad (1)$$

$$f_1^{Exploit} = PF_1 \times \frac{SeqCost_{Exploit}}{SeqTime} \quad (2)$$

- SeqCost: represents the cost improvement.
- SeqTime: execution time.
- PF1: weighting factor.

The second function f2. This function is used to evaluate the improvement in the objective function across three trials, allowing for the measurement of the stability of performance at each stage.

$$f_2^{explore} = PF_2 \times ((Seq1_{Cost\ Explore} + Seq2_{Cost\ Explore} + Seq3_{Cost\ Explore}))/ Seq1Time + Seq2Time + Seq3Time. \quad (3)$$

$$f_2^{Exploit} = PF_2 \times ((Seq1_{Cost\ Exploit} + Seq2_{CostExploit} + Seq3_{Cost\ Exploit}))/ Seq1Time + Seq2Time + Seq3Time. \quad (4)$$

Calculating the final evaluation score. The two resulting values are compared, and then the stage with the higher score is chosen to move to after the first rounds are completed.

$$Cost_{Explore} = PF_1 . f_1^{Explore} + PF_2 . f_2^{Explore} \quad (5)$$

$$Cost_{Exploit} = PF_1 . f_1^{Exploit} + PF_2 . f_2^{Exploit} \quad (6)$$

- Second: The Experienced Phase. After the initialization phase is complete, the algorithm becomes capable of making more precise decisions, and only one phase is selected in each iteration. The decision is based on three evaluation functions:

The first function f1. Is the instantaneous improvement, this function measures the amount of instantaneous improvement in the objective function during the current iteration.

$$f_1^t = PF_1 \times \frac{cost_{old}^{exploit} - cost_{new}^{exploit}}{T_{exploit}} \quad (7)$$

$$f_1^t = PF_1 \times \frac{cost_{old}^{explore} - cost_{new}^{explore}}{T_{explore}} \quad (8)$$

The second function, f2. Sequential improvement, this function aims to measure the continuity of improvement across several successive iterations.

$$f_2^{exploit} = PF_2 \times \frac{\sum_{K=1}^{3} cost_{old,k}^{exploit} - cost_{new,k}^{exploit}}{\sum_{K=1}^{3} T_{exploit,k}} \quad (9)$$

$$f_2^{explore} = PF_2 \times \frac{\sum_{K=1}^{3} cost_{old,k}^{explore} - cost_{new,k}^{explore}}{\sum_{K=1}^{3} T_{explore,k}} \quad (10)$$

The third function f3. Diversity, this function is used to promote diversity and prevent neglecting any stage for a long time is a randomly selected solution in the whole population.

$$f_3^t = \begin{cases} 0 \\ f_3^{t-1} + PF_3. \end{cases} \quad (11)$$

Final evaluation of the stage. The stage with the highest value is selected to continue the optimization process.

$$F_{Exploit}^t = a. f_{1exploit}^t + a. f_{2exploit}^t + a. f_{3exploit}^t \quad (12)$$

$$F_{explore}^t = a. f_{1explore}^t + a. f_{2explore}^t + a. f_{3explore}^t. \quad (13)$$

2.2 Exploration Phase

This equation represents the process of random jumping or exploration based on the difference in solutions.

$$Z_{i,G}=\begin{cases} RDim.(Ub-Lb)+Lb \\ X_a+G(X_a-X_b)+G(X_c-X_d)+(X_e-X_f \end{cases} \quad (14)$$

$X_a, X_b, X_c, X_d, X_e, X_f$: are solutions in the whole population that are aces are randomly selected.

Then the coefficient (G) is calculated using the equation:

$$G = 2.\mathrm{rand} - 1 \quad (15)$$

2.3 Exploitation Phase

In this phase, solutions are optimized using two strategies: sprinting and ambushing, as illustrated in the equation:

$$X_{new}=\begin{cases} mean(solutions) - X_r \\ pumamale + 2.rand.e^{randn}(X_r - X_i \end{cases} \quad (16)$$

Where:

- X_r: is a randomly selected solution in the whole population.
- X_i: is the current solution in the current iteration.

3 DERIVATION OF A NEW PARAMETER FOR THE CONJUGATE GRADIENT ALGORITHM

In 2024, Jiang proposed what is known as the spectral CG family, which introduces a spectral parameter into the update formula for the beta parameter β. Jiang based his work on the idea that the $M_k$hessian matrix can be approximated with a constant spectral number at the level of a single iteration [9].

$$Mk=\gamma KIk \quad (17)$$

$$yk=\gamma Ksk \quad (18)$$

Define the spectrometer by way of diagram:

$$\gamma_K = \frac{{y_k}^T s_k}{\|y_k\|^2} \quad (19)$$

Spectral residual error is as follows:

$$r_K = y_k - \gamma_K s_k \quad (20)$$

Jiang assumed that the new direction should be orthogonal to the spectral error.

$$ {d_{k+1}}^T r_K = 0 \quad (21)$$

$$ {d_{k+1}}^T (y_k - \gamma_K s_k) = 0 \quad (22)$$

In 2025, Chen developed the classical Hestenes-Stiefel (HS) method into spectral HS by directly inserting the γ_K spectrometer into the beta formula to fine-tune the numerical oscillation and improve the method's performance on large-scale problems [10].

$$\beta^{s\_sh} = \frac{{g_{k+1}}^T (y_k - \gamma_K s_k)}{{d_k}^T (y_k - \gamma_K s_k)} \quad (23)$$

- g_{k+1}:gradient vector.
- y_k: gradient difference vector.
- s_k:step vector.
- γ_K:spectral parameter.

Let's assume that:

$$y^* = r_K = y_k - \gamma_K s_k - \lambda_k g_{k+1} \quad (24)$$

- r_K: spectral residual vector;
- λ_k: step size.

Therefore, we also have the Quasi-Newtonian condition (QN) shown in the equation:

$$G_K s_k = y_K \quad (25)$$

Multiplying the equation by ${y_k}^T$, we get:

$$ {y_k}^T G s_k = {y_k}^T y_K \quad (26)$$

$$G = \frac{\|y_k\|^2}{{y_k}^T s_k}.I \quad (27)$$

$$G^{-1} = \frac{{y_k}^T s_k}{\|y_k\|^2}.I \quad (28)$$

$$ {d^N}_{k+1} = -G^{-1} g_{k+1} \quad (\mathrm{QN-direction}) \quad (29)$$

$$ {d^N}_{k+1} = -\frac{{y_k}^T s_k}{\|y_k\|^2}.I\, g_{k+1} \quad (30)$$

Multiplying the equation by ${y_k}^*$and using the stop condition $d_{k+1} r_k = 0$,We use $y^* = r_K$,and to simplify the derivation and obtain a more compact expression, we define the parameter ψ as follows:

$$\psi = \frac{{y_k}^T s_k}{\|y_k\|^2} \quad (31)$$

$$ {d^T}_{k+1} {y^*}_k = -\frac{{y_k}^T s_k}{\|y_k\|^2}.I\, {y^*}_k\, g_{k+1} \quad (32)$$

- ψ: spectral scaling.

$$ {d^T}_{k+1} {y^*}_k = -\psi\, {y^*}_k\, g_{k+1} = 0 \quad (33)$$

By definition:

$$d_{k+1} = -g_{k+1} + \beta_k s_k \quad (34)$$

Equation (34) defines the search direction update in the conjugate gradient algorithm.

- β_k:update parameter.

Multiplying (34) both sides of the equation by y_k^T.

$$d_{k+1} y_k^T = -y_k^T \ g_{k+1} + \beta_k y_k^T s_k \quad (35)$$

Equating (33) with (35) gives us:

$$-\psi y^*_k g_{k+1} = -y_k^T \ g_{k+1} + \beta_k y_k^T s_k \quad (36)$$

Substituting y^*_k from (24) into Equation (36) gives (37).

$$-\psi (y_k - \gamma_K s_k - \lambda_k g_{k+1})^T g_{k+1} = -y_k^T \ g_{k+1} + \beta_k y_k^T s_k \quad (37)$$

$$-\psi \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2 = -y_k^T \ g_{k+1} + \beta_k y_k^T s_k \quad (38)$$

$$-\psi \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2 + y_k^T \ g_{k+1} = \beta_k y_k^T s_k \quad (39)$$

$$(1-\psi) \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2 = \beta_k y_k^T s_k \quad (40)$$

The beta new:

$$\beta_{ZS}^{new} = \frac{(1-\psi) \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1}}{y_k^T s_k} + \frac{\psi \ \lambda_k \|g_{k+1}\|^2}{y_k^T s_k} \quad (41)$$

Where:

$0> \psi >1$, $0>\lambda_k>1$,
$y_k = g_{k+1} - g_k$, $s_k = x_{k+1} - x_k$.

Where (41) represents the final analytical expression of the proposed beta coefficient.

3.1 Proof of Sufficient radation

Theorem: the search direction d_{k+1} defined in (1) resulting from the conjugate gradient algorithm satisfies the property of sufficient regression for all values of K when the step length satisfies Wolff's conditions:

$$g^T_{k+1} d_{k+1} \le -M \|g_{k+1}\|^2 , \forall M > 0 \quad s_k \quad (42)$$

Proof:

Condition 1 will be proven using mathematical induction. When K = 0, then:

$$d_0 = -g_0 \ \rightarrow g^T_0 d_0 = -\|g_0\|^2 \quad (43)$$

Then the theorem is true. Now, let's assume the theorem is true for all values of K ≥1.

$$g^T_k d_k \le -M \|g_k\|^2 , \forall M > 0 \quad (44)$$

Let's assume we have:

$$d_{k+1} = -g_{k+1} + \beta_{new} d_k \quad (45)$$

We multiply both sides by g^T_{k+1}:

$$g^T_{k+1} d_{k+1} = -\|g_{k+1}\|^2 + \beta_{new} g^T_{k+1} d_k \quad (46)$$

We compensate for β_{new}:

$$\beta_{ZS}^{new} = \frac{(1-\psi) \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2}{y_k^T s_k} \quad (47)$$

$$g^T_{k+1} d_{k+1} = -\|g_{k+1}\|^2 + \left(\frac{(1-\psi) \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2}{y_k^T s_k}\right) \times g^T_{k+1} d_k \quad (48)$$

$$g^T_{k+1} d_{k+1} = -\|g_{k+1}\|^2 + \frac{g^T_{k+1} d_k}{y_k^T s_k} \left((1-\psi) \ y_k^T g_{k+1} + \psi \gamma_K s_k^T g_{k+1} + \psi \ \lambda_k \|g_{k+1}\|^2\right) \quad (49)$$

Wolf's strong conditions:

$$y^T_k s_k = \lambda_k \left(g^T_{k+1} d_k - g^T_k d_k\right) \ge \lambda_k \left((1-\sigma)\right) \left|g^T_k d_k\right| \quad (50)$$

$$g^T_{k+1} d_{k+1} \le -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\| \|d_k\|}{\lambda_k \left((1-\sigma)\right) \left|g^T_k d_k\right|} \left[(1-\psi) \ \|g_{k+1}\| \|y_k\| + \psi \gamma_K \ \|g_{k+1}\| \|d_k\| + \psi \ \lambda_k \|g_{k+1}\|^2\right] \quad (51)$$

$$g^T_{k+1} d_{k+1} \le -\|g_{k+1}\|^2 + \left[\frac{(1-\psi) \ \|y_k\| \ \|d_k\|}{\lambda_k \left((1-\sigma)\right) \left|g^T_k d_k\right|} + \frac{\psi \gamma_K \lambda_k \|d_k\|^2}{\lambda_k \left((1-\sigma)\right) \left|g^T_k d_k\right|} + \frac{\psi\sigma \ g_k^T \ d_k}{\lambda_k \left((1-\sigma)\right) \left|g^T_k d_k\right|}\right] \|g_{k+1}\|^2 \quad (52)$$

Where

$$\Omega = \frac{(1-\psi) \ \|y_k\| \ \|d_k\|}{\lambda_k ((1-\sigma)) \left|g^T_k d_k\right|} + \frac{\psi \gamma_K \lambda_k \|d_k\|^2}{\lambda_k ((1-\sigma)) \left|g^T_k d_k\right|} + \frac{\psi\sigma \ g_k^T \ d_k}{\lambda_k ((1-\sigma)) \left|g^T_k d_k\right|} \quad (53)$$

$$g^T{}_{k+1}d_{k+1} = -\|g_{k+1}\|^2 + \Omega\|g_{k+1}\|^2 \tag{54}$$

$$g^T{}_{k+1}d_{k+1} = -(1-\Omega)\|g_{k+1}\|^2 \tag{55}$$

3.2 Proof of the Global Convergence Analysis for the Conjugate Gradient Algorithm

Theorem: assume that the hypothesis is true, that the conjugate gradient method is true in the direction of the search slope d_k, and that the step length $\lambda_k$is generated based on the swp conditions, then

$$\lim_{K\to\infty} inf\,\|g_k\| = 0 \tag{56}$$

$$\sum_{K=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} = \infty \tag{57}$$

$$d_{k+1} = -g_{k+1} + \beta_{new}d_k \tag{58}$$

We take the norm for both sides

$$\|d_{k+1}\| = \|-g_{k+1} + \beta_{new}d_k\| \tag{59}$$

$$\|d_{k+1}\| \le \|-g_{k+1}\| + |\beta_{new}|\|d_k\| \tag{60}$$

$$\|d_{k+1}\| \le \|-g_{k+1}\| + \left|\frac{(1-\psi)\ y_k{}^T g_{k+1} + \psi\,\gamma_K\,s_k{}^T g_{k+1} + \psi\ \lambda_k\|g_{k+1}\|^2}{y_k{}^T\,s_k}\right|\|d_k\| \tag{61}$$

$$|\beta^{New}\ | = \left|\frac{(1-\psi)\ y_k{}^T g_{k+1} + \psi\,\gamma_K\,s_k{}^T g_{k+1} + \psi\ \lambda_k\|g_{k+1}\|^2}{y_k{}^T\,s_k}\right| \tag{62}$$

$$|\beta^{New}| \le \left|\frac{(\,1.2 - \ 1.2\psi)\|g_{k+1}\|^2 - \sigma\psi\,\gamma_K\lambda_k\,C\|g_k\|^2 + \psi\ \lambda_k\|g_{k+1}\|^2}{-(1+\sigma)\lambda_k C\|g_k\|^2}\right| \tag{63}$$

$$|\beta^{New}| \le \left|\frac{1}{\|g_k\|^2}\left[\frac{\begin{bmatrix}(\,1.2 - \ 1.2\psi)\\ +\psi\ \lambda_k\end{bmatrix}\|g_{k+1}\|^2}{-(1+\sigma)\lambda_k C} - \frac{\sigma\psi\,\gamma_K\,C\|g_k\|^2}{-(1+\sigma)C}\right]\right| \tag{64}$$

$$|\beta^{New}\ | = F \tag{65}$$

$$\|d_{k+1}\| \le \|g_{k+1}\| + |\beta_{new}|\|d_k\| \tag{66}$$

$$\|d_{k+1}\| \le \|g_{k+1}\| + F\|d_k\| \tag{67}$$

$$\|d_{k+1}\| \le U + FD \le M \tag{68}$$

$$\frac{1}{\|d_{k+1}\|} \ge \frac{1}{M} \tag{69}$$

$$\sum_{K=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \ge \sum_{K=1}^{\infty} \frac{1}{M^2} = \frac{1}{M^2}\sum_{K=1}^{\infty} 1 = +\infty \tag{70}$$

$$\lim_{K\to\infty} inf\,\|g_k\| = 0 \tag{71}$$

4 THE PROPOSED HYBRID ALGORITHM (DRA-CG-PO)

4.1 General Concept

The proposed algorithm is based on a hybrid of the DRA-CG algorithm and the PO algorithm to improve the overall performance of the core algorithm, particularly in achieving a better balance between global search and local exploitation. In this context, DRA-CG serves as the core optimization algorithm, combining the global search capabilities of the (DRA) [5], with the high-speed local optimization efficiency of the CG method [11]. However, the performance of DRA-CG can be affected in some complex problems by low diversity or early confinement to limited areas of the search space. Therefore, the PO algorithm is integrated with DRA-CG to enhance global exploration and expand the search space [12]. PO is used to guide the search towards more promising areas, while DRA-CG handles the task of optimizing solutions locally with high accuracy. This is how CG avoids using a random starting point; it starts at a well-selected and optimized solution. This lowers the risk of finding suboptimal solutions in the local area and therefore improves the quality of the optimal solution [13], [14].

4.2 Hybrid Algorithm Structure

The general structure of the proposed hybrid algorithm (PO-DRA-CG) can be summarized as follows as shown in Figure 1:

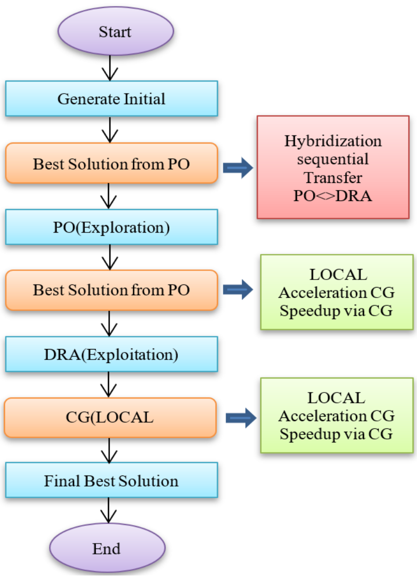

Figure 1: Structure of the proposed hybrid algorithm (PO-DRA-CG).

5 NUMERICAL RESULTS

In this section, the performance of the proposed hybrid algorithm (PO-DRA-CG) was evaluated using ten standard test functions selected from a set of twenty-three reference test functions (F1-F23). The selected functions included both unimodal and multimodal functions to assess the algorithm's ability to balance exploration and exploitation during the optimization process.

The problem dimension was set to D = 30, with a population size of 50, and a maximum of 1000 iterations. To ensure the fairness of the comparison and the reliability of the results, all algorithms were run independently ten times using the same settings.

The performance of the proposed algorithm was also compared with the original Puma algorithm (PO), the Divine Religion algorithm (DRA), and the DRA-CG hybrid algorithm.

The results, presented in Table 1, show that the proposed algorithm (PO-DRA-CG) achieved superior performance in terms of solution accuracy and convergence stability compared to the other algorithms. Furthermore, the convergence behavior of the algorithms is illustrated in Figures 2-5, where it can be observed that the proposed hybrid algorithm converges more rapidly and avoids premature entrapment in local optima, thereby demonstrating a better balance between exploration and exploitation. Overall, the results confirm that integrating the PO algorithm with the DRA-CG framework leads to a significant improvement in both performance and efficiency. It is worth noting that the DRA-CG value reported for F5 (-2.89997e-317) is not exactly zero but is several orders of magnitude below machine-precision-relevant thresholds; this reflects floating-point underflow near the optimum rather than a distinct numerical outcome, and is effectively equivalent to the zero values reported for the other algorithms on this function.

Analysis of convergence curvesFigures 2-5 illustrate the convergence behavior of the various algorithms on the test function F7. It is observed that the proposed algorithm (PO-DRA-CG) achieves a faster convergence rate and lower final values compared to the other algorithms. This indicates an improvement in search efficiency resulting from the integration of the exploration mechanisms within the PO algorithm and the exploitation capabilities of DRA-CG. Furthermore, this performance reflects the role of the proposed coefficient β in enhancing the stability of the search direction and improving convergence characteristics.

Table 1: Comparing results between the PO algorithm, DRA-CG hybrid algorithm and the PO-DRA-CG hybrid algorithm using a population size of 50 and 1000 iterations

Fun	PO	DRA	DRA-CG	DRA-CG-PO
F_1	0	0	0	0
F_2	1.274e-267	0	0	0
F_3	0	0	0	0
F_4	8.25979e-270	0	0	0
F_5	25.3422	1.2898e-05	-2.89997e-317	0
F_6	8.6710e-10	0	0	0
F_7	2.96411e-05	3.4243e-07	5.24039e-228	0
F_8	-12569.5	-10423.5	-10457.3	-12054.3
F_9	0	0	0	0
F_{10}	4.4408e-16	4.4408e-16	2.791e-204	0

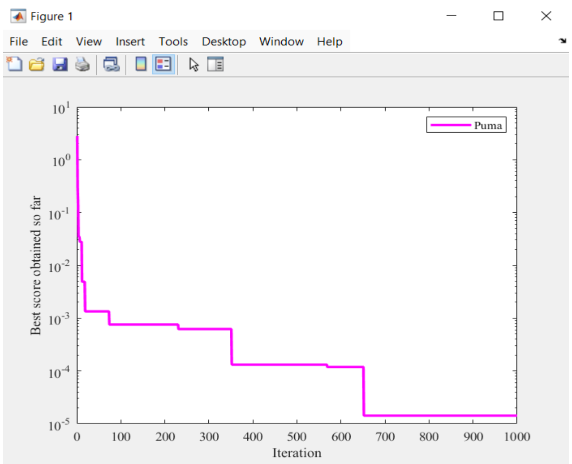

Figure 2: Convergence curve of the puma algorithm on benchmark function F7.

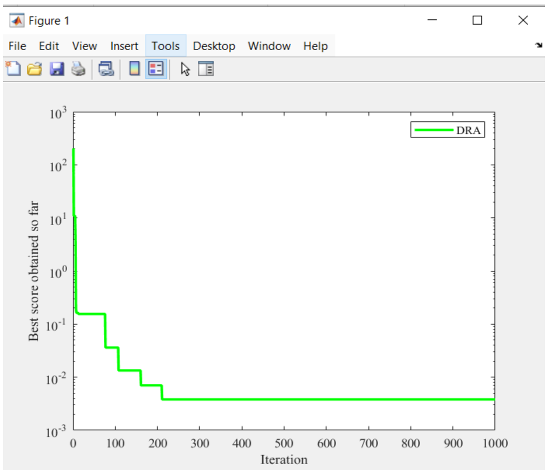

Figure 3: Convergence curve of the DRA algorithm on benchmark function F7.

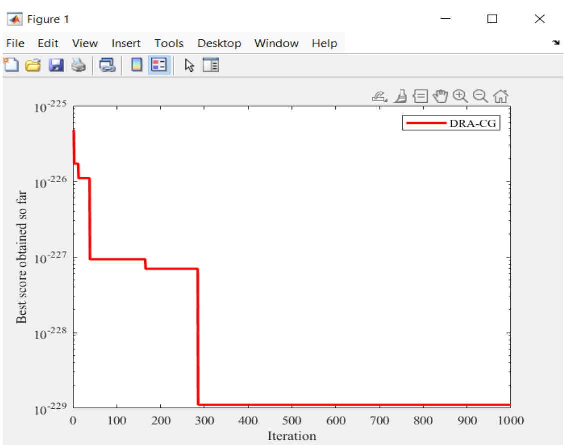

Figure 4: Convergence curve of the DRA- CG algorithm on benchmark function F7.

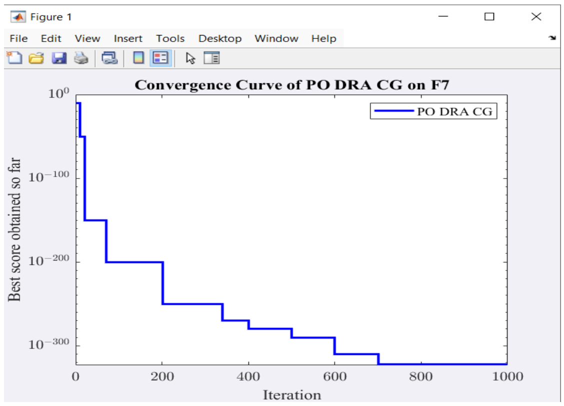

Figure 5: Convergence curve of the PO-DRA-CG algorithm on benchmark function F7.

6 CONCLUSIONS

This research developed an enhanced hybrid optimization method that combines effective global exploration with high-precision local search within a unified framework, thereby achieving a balance between exploration and exploitation for continuous optimization problems. Experimental results demonstrated that the proposed method achieved higher solution quality, faster convergence, and greater stability than the PO, DRA, and DRA-CG algorithms across most benchmark functions, including both unimodal and multimodal test problems. The only exception was the multimodal function F8, for which the original PO algorithm produced a marginally better result than the proposed hybrid (Table 1). These findings indicate that the proposed hybrid approach improves solution quality and robustness for the majority of benchmark problems while maintaining competitive performance on more challenging multimodal functions. Furthermore, the hybrid architecture exhibits an enhanced ability to escape local optima and achieve more accurate solutions near the global optimum, making it a promising approach for practical optimization and engineering applications where both reliability and computational efficiency are essential.

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