Bayesian Approach of Birnbaum-Saunders Quantile Regression via Jeffreys nd Weakly Informative Priors

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Abstract: In this article we propose a general Bayesian estimation framework for the Birnbaum-Saunders Quantile Regression (BSQ) model which is used to analyze asymmetric and heterogeneous economic data. In this work, the novel aspect is the joint incorporation of several existing surrogate structures (with different degrees of approximation) and decision-theoretic loss functions (with direct impact on estimation performance trade-offs) under a common BSQ framework conducive to systematic testing of their effect on quantile estimation performance. We consider two previous prior specifications which are Jeffreys prior and weakly informative priors together with three loss functions quadratic, LINEX and the proposed quadratic-logarithmic loss function. Iraqi economic data analysis demonstrates significant disparities in independent variable impacts across quantiles, confirming heterogeneous distribution. The results show further that Jeffreys prior yields more consistent estimates in most quantiles, whereas weakly informative priors give better coverage under the tail. Also, this highlights the importance of choosing a loss function and motivates asymmetric and hybrid loss functions for improved accuracy, especially in extreme quantiles. This contribution expands upon previous works by introducing a new general comparative framework that overextends the potential of Bayesian BSQ modelling, providing robustness and flexibility in such context with significant practical function resulting estimation improves in economic real-world applications.

1 INTRODUCTION

Classical regression models [1], [2] approach that only relies on estimation of conditional mean is not working well when the correlation structure in economic datasets goes beyond simple linearity and includes skewness, heterogeneity and departures from normality. It has a lot of economic implications as the effects of explanatory variables can vary across the conditional distribution of the response. A mean-based regression model formulation may not be sufficiently broad enough to explain the underlying socio-economic relationships by themselves.

This limitation could be addressed by using quantiles because quantile regression can characterize the effect of covariates at various quantiles of the conditional distribution [3]. It can explain heterogeneous relations without any assumption, which is especially well suited when dealing with economic data that are often asymmetrical and heteroscedastic.

The Birnbaum-Saunders distribution has received much attention for modeling positive skewed data due to its flexibility in modeling asymmetric behavior given the context of economic and reliability studies [4], [5]. This distribution combined with quantile regression leads to the Birnbaum-Saunders Quantile Regression (BSQ) model which directly estimates conditional quantiles preserving the distributional properties of skew responses [6].

In the Bayesian framework, estimating a parameter lets you map orientative knowledge, while it produces a full probabilistic description of model parameters [7]. Our standard Bayesian BSQ models, which are widely taught and used in practice, face a problem of dependence between the ultimate performance of the model on the prior distributions & loss functions choosen to encourage stability but also accuracy of the estimators. While we recognize that there is more literature in this domain, afaik most existing studies either focus on prior specification or loss functions (at least faculties but rarely all together) with little emphasis regarding their joint effect across quantile levels at which interests.

This presents a clear research gap of having, from the empirical point of view on more real economic data featuring its characteristics skewedness and heterogeneity, a wider range of prior distributions and loss functions in, respectively a Bayesian BSQ model.

In order to bridge this gap, the current work addresses Bayesian estimation of the BSQ model under both two prior specifications (Jeffreys prior and weakly informative priors) together with three loss functions (Quadratic, LINEX, and Quadratic-Logarithmic). Because we calibrate the model to actual economic data, we can use this to explore the effects on estimation quality (at several quantiles) of both previous choices as well as loss functions.

This article contributes by providing a systematic comparison of existing structures and loss functions within the BSQ approach, along with empirical analysis regarding their joint functional impact on modeling heterogeneous economic relations through the distribution.

2 BACKGROUND AND MOTIVATION

Classical regression techniques, for instance, are focused on the conditional mean of Y given X where errors are normally distributed. In practice, however, economic data are very rarely normal and the presence of skewness and heavy tails in addition to heteroscedasticity means that mean-based methods are heavily restricted [1], [2].

Fiscal distinctions: these are all economic variables whose densities (are asymmetric) and where the effect of governed variables can be conditional whether agents sit on various parts of that distribution. That is, old-school models might overlook important nuances in economic relations.

This is concluded as quantile regression provide relative to ordinary least square system where have more additional worth or points from suggested yet adjust their linear toward numerous place [3] form setting are addressed by specific angle ideas particularly unexplored information outer segment, heterogeneous datasets.

Particularly, the Birnbaum-Saunders distribution is a suitable candidate for the modeling of positively skewed variables [4], [5]; and its contemplation in combination with quantile regression (the BSQ model), which allows conditional quantiles to be modeled directly while keeping fundamental features of the distribution [6].

Providing a probabilistic interpretation of the parameter estimates [7]. This Article examines Bayesian BSQ regression with different prior specifications and loss functions in a flexible manner capable of fitting economic data exhibiting skewness and homogeneity.

3 BIRNBAUM-SAUNDERS DISTRIBUTION (BS)

The Birnbaum-Saunders (BS) distribution proposed by Birnbaum and Saunders (1969) is one of the most successful distributions for positively supported right-skewed data. This distribution can be obtained from transformation [4], [5] of a standard normal random variable.

Let:

$$Z \sim N(0,1), \tag{1}$$

and define the transformation:

$$T = \frac{\beta}{4}\left(\alpha Z + \sqrt{\alpha^2 Z^2 + 4}\right)^2. \tag{2}$$

Then, the random variable T follows a Birnbaum-Saunders distribution, denoted by:

$$T \sim BS(\alpha, \beta), \tag{3}$$

where:

- $\alpha > 0$ is the shape parameter;
- $\beta > 0$ is the scale parameter.

The probability density function of the BS distribution is:

$$f_T(t) = \frac{1}{2\alpha\beta\sqrt{2\pi}}\left(\sqrt{\frac{\beta}{t}} + \sqrt{\frac{\beta^3}{t^3}}\right)\exp\left[-\frac{1}{2\alpha^2}\left(\sqrt{\frac{t}{\beta}} - \sqrt{\frac{\beta}{t}}\right)^2\right], t > 0. \tag{4}$$

The cumulative distribution function is given by:

$$F_T(t) = \Phi\left[\frac{1}{\alpha}\left(\sqrt{\frac{t}{\beta}} - \sqrt{\frac{\beta}{t}}\right)\right], \quad t > 0\,, \tag{5}$$

where $\Phi(\cdot)$denotes the standard normal cumulative distribution function.

The first two moments of the distribution are:

$$E[T] = \beta\left(1 + \frac{\alpha^2}{2}\right). \tag{6}$$

$$\mathrm{Var}[T] = \beta^2\alpha^2\left(1 + \frac{5}{4}\alpha^2\right). \tag{7}$$

The BS distribution is defined for positive values (t>0) and is commonly used for modelling asymmetric and positively skewed data.

3.1 Birnbaum-Saunders Distribution Parametrized by Its Quantiles

Quantiles are useful as measures of distributional behaviour of random variables, especially in the case of skewed data. For a level $q \in (0,1)$, the q -th quantile is defined as the value below which a proportion q of observations falls. Because the Birnbaum-Saunders (BS) distribution can be expressed as a transformation of the standard normal distribution, its quantiles can then be computed directly from their corresponding normal ones [8].

Define the standard normal quantile by:

$$z_q = \Phi^{-1}(q), \tag{8}$$

where $\Phi^{-1}(\cdot)$denotes the inverse cumulative distribution function of the standard normal distribution. The q -th quantile of the BS distribution can then be written as:

$$Q = \frac{\beta}{4}\left(\alpha z_q + \sqrt{\alpha^2 z_q^2 + 4}\right)^2 = \frac{\beta}{4}\gamma_\alpha^2 , \tag{9}$$

and define:

$$\gamma_\alpha = \alpha z_q + \sqrt{\alpha^2 z_q^2 + 4} . \tag{10}$$

Using this relationship, the scale parameter β can be expressed in terms of the quantile Q, allowing the BS distribution to be reparameterized in terms of (α,Q).

Under this parameterization, the cumulative distribution function is:

$$F(t) = \Phi\left[\frac{1}{\alpha\gamma_\alpha}\sqrt{\frac{4Q}{t}}\left(\frac{t\gamma_\alpha^2}{4Q} - 1\right)\right], \quad t > 0 . \tag{11}$$

The corresponding probability density function is

$$f_T(t) = \frac{1}{\alpha\gamma_\alpha\sqrt{8\pi Q t}}\left(\gamma_\alpha^2 + \frac{2Q}{t}\right)\exp\left[-\frac{2Q}{\alpha^2\gamma_\alpha^2 t}\left(\frac{t\gamma_\alpha^2}{4Q} - 1\right)^2\right], \quad t > 0 . \tag{12}$$

Under the quantile parameterization, the BS distribution is characterized by two positive parameters: the shape parameter $\alpha > 0$and the quantile parameter $Q > 0$, where$P(T \leq Q) = q$.

3.2 Birnbaum–Saunders Quantile Regression Model

The Birnbaum-Saunders quantile regression model is obtained by reparametrizing the Birnbaum-Saunders distribution in terms of a fixed quantile level $q \in (0,1)$ [9].

Let:

$$T_i \sim BS(\alpha, Q_i), \quad i = 1, \dots, n, \tag{13}$$

where:

- $Q_i > 0$ is the conditional quantile;
- $\alpha > 0$ is the shape parameter.

This formulation follows from the one-to-one transformation between the classical parameterization (α, β) and the quantile-based parameterization (α, Q).

To incorporate explanatory variables, the conditional quantile is linked to a linear predictor through a link function similar to generalized linear models:

$$h(Q_i) = \eta_i = x_i^\top \beta, \ i = 1, \dots, n , \tag{14}$$

where:

- $x_i = (1, x_{i1}, \dots, x_{i(p-1)})^\top$ is the vector of explanatory variables,
- $\beta = (\beta_0, \dots, \beta_{p-1})^\top$ is the vector of regression coefficients.

Hence, the conditional quantile is expressed as:

$$Q_i = h^{-1}(x_i^\top \beta) . \tag{15}$$

This formulation enables direct modelling of conditional quantiles while preserving the positive support and right-skewed structure of the Birnbaum-Saunders distribution.

4 BAYESIAN ESTIMATION FRAMEWORKS

Bayesian estimation provides a coherent framework for updating prior beliefs with observed data, treating parameters as random variables with associated probability distributions [7].

This approach formally combines prior information and sample evidence within a unified structure.

4.1 Likelihood Function

Given a parameter vector $\theta = (\beta^\top, \alpha)^\top$ and observed data $y = (y_1, \dots, y_n)$ with conditional density $f(y_i \mid \alpha, Q_i)$, the likelihood function:

$$L(\theta \mid y) = \prod_{i=1}^{n} f\,(y_i \mid \alpha, Q_i) , \tag{16}$$

quantifies the support that the data provides for different parameter [7].

4.2 Prior Specification

Prior distributions in Bayesian analysis express beliefs about parameters before observing data.

This article uses both the non-informative Jeffreys prior and weakly informative priors to stabilize estimation without imposing strong assumptions.

4.2.1 Jeffreys Prior

The Jeffreys prior is a non-informative prior in Bayesian statistics, derived from the Fisher information matrix to provide a parameterization-invariant prior distribution. It is widely used when little or no prior information is available, allowing the data to primarily determine the posterior distribution. Jeffreys prior is defined as [10]:

$$\pi_J(\theta) \propto \sqrt{\det\left(K_{\theta\theta}(\theta)\right)}, \quad (17)$$

$$\det(K_{\theta\theta}) = \left|K_{\beta\beta}\right| \left|K_{\alpha\alpha} - K_{\alpha\beta}K_{\beta\beta}^{-1}K_{\beta\alpha}\right|. \quad (18)$$

Therefore,

$$\pi_J(\theta) \propto |K_{\beta\beta}|^{\frac{1}{2}} \left|K_{\alpha\alpha} - K_{\alpha\beta}K_{\beta\beta}^{-1}K_{\beta\alpha}\right|^{\frac{1}{2}}, \quad (19)$$

where:

- $\pi_J(\theta)$ denotes the Jeffreys prior distribution.
- $K_{\theta\theta}(\theta)$ represents the Fisher information matrix.
- $\det(\cdot)$ denotes the determinant of the matrix.

The Fisher information matrix is defined as:

$$K_{\theta\theta}(\theta) = -E\left[\frac{\partial^2 \log L(\theta)}{\partial\theta\,\partial\theta^\top}\right], \quad (20)$$

where $L(\theta)$ is the likelihood function of the model.

This represents the full Jeffreys prior for the BSQ regression model.

4.2.2 Weakly Informative Priors

Weakly informative priors offer a mechanism for regularization in Bayesian analysis [11], providing sufficient information to stabilize parameter estimates without causing strong influence on the results. In the BSQ model, regression coefficients are given normal priors with some uncertain distance from 0 and the shape parameter is assigned a half-Cauchy prior allowing data to inform inference but preventing retrospective catastrophe.

$$\beta \sim N_p\left(0, \sigma_\beta^2 I_p\right),$$

$$\alpha \sim \text{Half-Cauchy}(0, \sigma_\alpha), \alpha > 0 \quad (21)$$

Under these assumptions, the joint prior distribution can be written as

$$\pi(\beta, \alpha) = \pi(\beta)\pi(\alpha) \ . \quad (22)$$

4.3 Posterior Distribution

In Bayesian inference, the posterior distribution combines prior information about the parameters with data evidence via the likelihood function. By Bayes' rule [7], the posterior for the parameter vector $\theta = (\beta^\top, \alpha)^\top$ is given by:

$$\pi(\theta \mid t) \propto L(\theta \mid t)\,\pi(\theta), \quad (23)$$

where $L(\theta \mid t)$ is the likelihood for the Birnbaum-Saunders Quantile (BSQ) model and $\pi(\theta)$is the prior. The posterior summarizes all available information about the parameters after observing the data and forms the basis for Bayesian estimation and inference. Depending on the prior specification, the posterior can be written as follows.

4.3.1 Posterior Distribution under Jeffreys Prior

With the Jeffreys prior, the prior is defined using the Fisher information matrix [10]:

$$\pi_J(\theta) \propto \sqrt{|K_{\theta\theta}(\theta)|}, \quad (24)$$

yielding the posterior:

$$\pi(\theta \mid t) \propto L(\theta \mid t)\sqrt{|K_{\theta\theta}(\theta)|}\ . \quad (25)$$

This non-informative prior lets the data primarily determine the posterior.

4.3.2 Posterior Distribution under Weakly Informative Priors

For weakly informative priors, limited prior information is introduced to stabilize estimation [11]:

$$\beta \sim N_p\left(0, \sigma_\beta^2 I_p\right),$$

$$\alpha \sim \text{Half-Cauchy}(0, \sigma_\alpha), \alpha > 0\ . \quad (26)$$

The posterior then becomes:

$$\pi(\beta, \alpha \mid t) \propto L(\beta, \alpha \mid t)\,\pi(\beta)\,\pi(\alpha)\ . \quad (27)$$

This approach allows the data to drive inference while preventing implausible parameter values.

4.4 Bayesian Estimation under Three Loss Functions

In the Bayesian framework, parameter estimation is based on minimizing the posterior risk associated with a chosen loss function [12]. Given the posterior distribution $\pi(\theta|t) \propto L(\theta|t)\pi(\theta)$, where $L(\theta|t)$ is the likelihood for the BSQ model and $\pi(\theta)$ is the prior (either Jeffreys or weakly informative), the optimal Bayesian estimator minimizes the posterior expected loss:

$$\rho(\delta|t) = E[L(\theta,\delta)|t] = \int L(\theta,\delta)\pi(\theta|t)d\theta. \quad (28)$$

Three loss functions are considered in this article.

4.4.1 Quadratic Loss

The quadratic loss function penalizes positive and negative estimation errors equally:

$$L(\theta,\delta) = (\theta-\delta)^2. \quad (29)$$

The Bayesian estimator under quadratic loss is the posterior mean.

$$\hat{\theta}_Q = E(\theta|t) = \int \theta\pi(\theta|t)d\theta. \quad (30)$$

4.4.2 Linex Loss

The Linex loss function accommodates asymmetric error penalties:

$$L(\theta,\delta) = \exp\big(a(\delta-\theta)\big) - a(\delta-\theta) - 1. \quad (31)$$

The Bayesian estimator is:

$$\hat{\theta}_{LINEX} = -\frac{1}{a}\ln\big(E\big[e^{-a\theta}\big|t\big]\big)\,. \quad (32)$$

4.4.3 Proposed Quadratic-Logarithmic Loss

The proposed quadratic-logarithmic loss combines quadratic and logarithmic components for greater robustness.

$$L(\theta,\delta) = \omega(\theta-\delta)^2 + (1-\omega)\left[\log\left(\frac{\theta+\varepsilon}{\delta+\varepsilon}\right)\right]^2. \quad (33)$$

Where:

- $(0 < \omega < 1)$ balances the two terms;
- $(\varepsilon > 0)$ ensures numerical stability.

The Bayesian estimator minimizes the posterior risk.

$$\hat{\theta}_{QL} = \underset{\delta}{\operatorname{argmin}} \int L(\theta,\delta)\pi(\theta|t)d\theta. \quad (34)$$

5 ECONOMIC DATA APPLICATION

This section presents an empirical application of the proposed Bayesian Birnbaum–Saunders Quantile (BSQ) model using economic data from Iraq. The purpose of this application is to demonstrate the capability of the proposed methodology to model asymmetric economic responses and capture heterogeneous effects across different quantiles of the dependent variable.

The analysis focuses on private consumption behavior and investigates the effects of key macroeconomic factors, including price dynamics and population changes. The application provides an empirical validation of the proposed Bayesian framework using real economic observations.

5.1 Data Description and Model Specification

The empirical study is based on annual economic data obtained from the Iraqi Ministry of Planning covering the period between the 1990s and the 2000s. The dataset consists of 33 observations with a time-series structure.

Although the sample size is relatively small, it remains suitable for Bayesian analysis because prior information can improve parameter stability and provide more reliable estimation when the available observations are limited.

The response variable is defined as:

$$Y = \frac{\text{Private Consumption}}{\text{GDP}},$$

which represents the proportion of economic activity associated with private consumption.

The explanatory variables included in the model are:

- X_1: implicit GDP deflator;
- X_2: consumer price index (CPI);
- X_3: population size.

These variables were selected to capture both price-related effects and aggregate demand characteristics that influence consumption behavior.

The Birnbaum–Saunders distribution was adopted because of its ability to represent positively skewed data. The goodness-of-fit evaluation produced a test statistic:

$$D = 0.1293,$$

with the estimated shape parameter:

$$\hat{\alpha} = 0.3629 < 1.$$

These results indicate an adequate model fit with moderate right-skewness, which is consistent with the characteristics of the economic data.

The obtained findings agree with previous economic research showing that private consumption is a major contributor to economic growth, while inflationary effects may reduce purchasing power [13]. Therefore, the Bayesian BSQ model provides an appropriate framework for analysing skewness and heterogeneous relationships across different quantiles of the response variable.

5.2 Model Estimation

To investigate the differential effects of the explanatory variables across the conditional distribution of the response variable, the Bayesian Birnbaum–Saunders Quantile (BSQ) model was estimated at multiple quantile levels: $\tau = 0.10, 0.25, 0.50, 0.75$ and 0.90

These quantile levels were selected to represent the lower, median, and upper regions of the conditional distribution, allowing the analysis of heterogeneous economic effects beyond the average response.

Bayesian parameter estimation was performed using two alternative prior specifications:

1) Jeffreys prior;
2) weakly informative prior.

For each prior specification, the model parameters were estimated under three different loss functions:

- quadratic loss function;
- LINEX loss function;
- combined quadratic-logarithmic loss function.

This estimation framework allows a comprehensive evaluation of how prior assumptions and loss functions influence parameter estimates and uncertainty quantification.

The estimated parameters obtained from these approaches are presented and discussed in the following section.

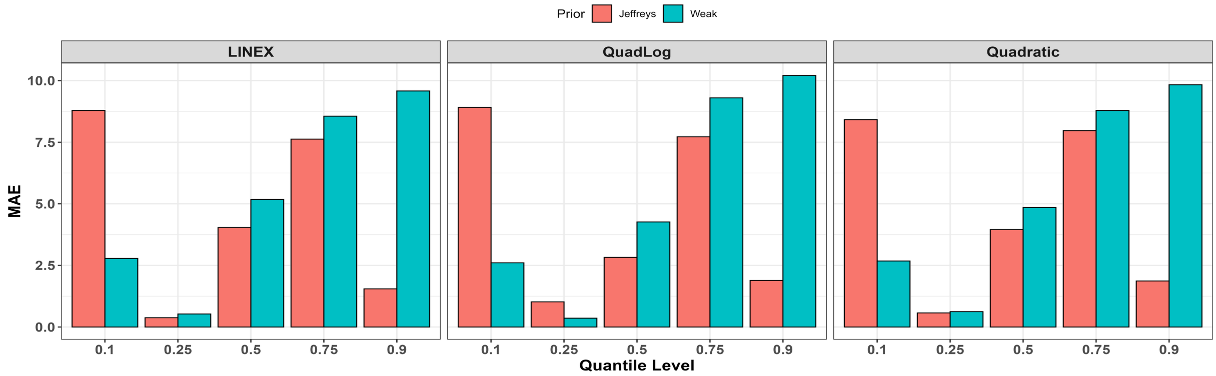

Figure 1: Comparison of jeffreys and weak priors based on MAE.

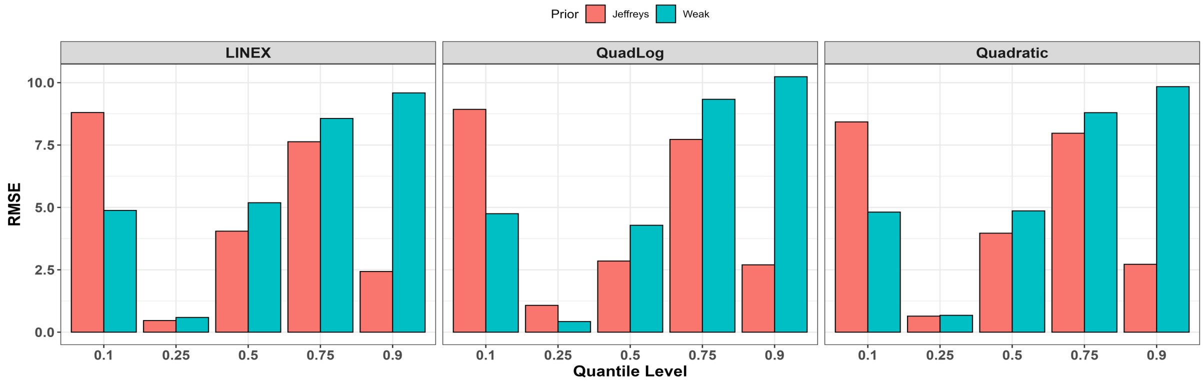

Figure 2: Comparison of jeffreys and weak priors based on RMSE.

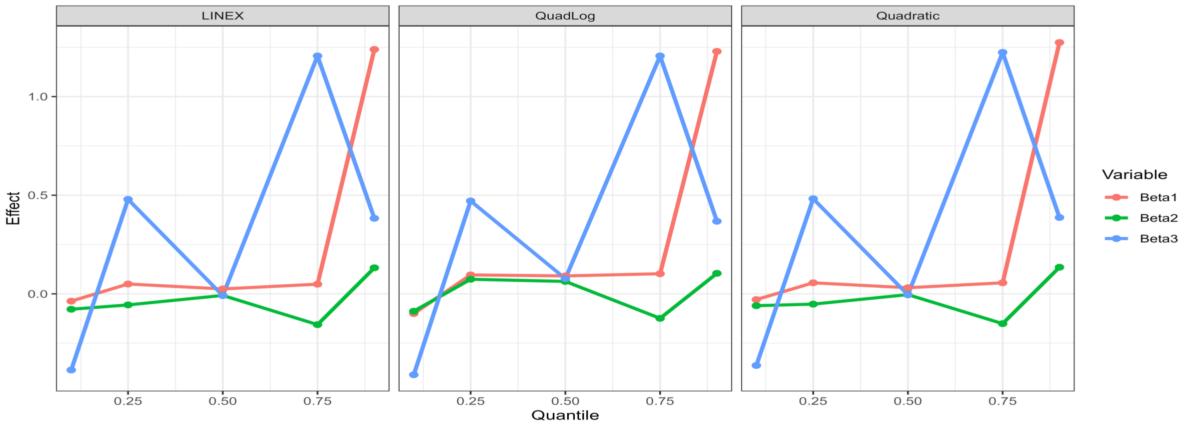

Figure 3: Effect of economic variables.

6 RESULTS AND DISCUSSION

As can be seen from the comparison of mean absolute error (MAE) performance Figure 1.

This clearly shows the patterns of variation in MAE along the distribution, comparing across quantiles (left to right, top to bottom) and different loss functions. The weakly informative prior leads to lower MAE results (smaller value) for nearly all loss functions and quantiles $\tau = 0.10$ and $\tau = 0.25$ in the lower part of the distribution, indicating improved estimation precision on that segment of quantiles reflecting the lower tail behavior of the conditional distribution. Suggesting that the weak prior is more permissive of low consumption-to-GDP ratios.

However, this trend is reversed at higher quantile level. Jeffreys prior has lower MAE values than the non-informative prior at these quantiles, indicating, on behalf of median and upper quantiles ($\tau =$ 0.50,0.75,0.90), a better estimation accuracy for central part and stability in width of distribution of data. The numerical evidence suggests that LINEX and our proposed quadratic-logarithmic loss function improve performance to a higher quantile level and hence recommend attention in choosing a loss function to achieve improvements in estimation efficiency. In short, the findings shown in Figure 1 indicate that although the weakly informative prior performs better at lower quantiles, across most of the distribution, the Jeffreys prior provides more reliable estimates, as shown in Table 1.

Findings in Figure 2 (based on RMSE) do corroborate these results.

The weakly informative prior performs better (lower RMSE) particularly at the lower quantiles $\tau = 0.10$ and $\tau = 0.25$ demonstrating how well it models the lower tail of residual distributions relative to their observed counterparts. But we can see, as the quantile level rises, that the Jeffreys prior definitively dominates. Overall, at $\tau = 0.50, 0.75$ and 0.90 it produces generally lower RMSE values than the majority of loss functions tested implying that this method is far more robust which in turn provides a less sensitive estimated model as shown in Table 2.

Table 1: Better MAE-loss in quantiles than others: full MAE losses.

Quantile	Best Loss Function	Best MAE	Prior Type
0.10	Quad-Log	2.6040	Weakly Informative
0.25	LINEX	0.3750	Jeffreys
0.50	Quad-Log	2.8264	Jeffreys
0.75	LINEX	7.6259	Jeffreys
0.90	LINEX	1.5477	Jeffreys

Table 2: Optimal Loss function selection across quantiles based on minimum RMSE.

Quantile	Best Loss Function	Best RMSE	Prior Type
0.10	Quad-Log	4.7486	Weakly Informative
0.25	Quad-Log	0.4259	Weakly Informative
0.50	Quad-Log	2.8512	Jeffreys
0.75	LINEX	7.6321	Jeffreys
0.90	LINEX	2.4313	Jeffreys

It appears that the quadratic-logarithmic loss function performs particularly well at intermediate quantile levels, whereas the LINEX loss function provides better performance at more extreme quantiles.

Combined with the results presented in Figures 1 and 2, these findings demonstrate that relying on a single evaluation criterion may provide an incomplete assessment of model performance. The simultaneous consideration of MAE and RMSE provides a more comprehensive evaluation, as MAE measures average estimation errors while RMSE gives greater importance to larger deviations. The consistency of both evaluation criteria supports the conclusion that the Jeffreys prior generally provides more robust and efficient estimates, particularly outside the lower tail of the distribution.

Based on these results, the economic interpretation of the regression coefficients is performed using the Bayesian BSQ model estimated with the Jeffreys prior.

As shown in Figure 3, the estimated regression coefficients $\beta_1, \beta_2, \beta_3$vary across quantile levels, indicating heterogeneous economic effects. Since the dependent variable represents the ratio of private consumption to GDP, the coefficients should be interpreted as changes in the consumption share of total economic output rather than changes in absolute consumption levels.

For the GDP deflator coefficient (β_1), the estimated effect is negative or weak at lower quantiles but becomes positive and stronger at higher quantile levels. This indicates that the relationship between price levels and the consumption-to-GDP ratio is not uniform across the distribution. At higher consumption-share levels, increases in the general price level may be associated with a higher consumption ratio, whereas the effect remains limited or negative at lower quantiles. This result confirms the heterogeneous relationship between economic variables across the conditional distribution, which is consistent with the principles of quantile regression theory discussed in [1].

The coefficient of the consumer price index (β_2) is predominantly negative across most quantile levels, with stronger negative effects observed in the lower and middle quantiles. This suggests that inflation may reduce the consumption-to-GDP ratio through the erosion of purchasing power. This finding is consistent with economic theory and previous Bayesian quantile regression studies, such as [2].

For the population size variable (β_3), the estimated coefficient is weak or negative at lower quantiles but becomes positive and significant at higher quantile levels. This suggests that population growth is associated with an increased consumption-to-GDP ratio in economies with higher consumption shares. This relationship reflects the role of population expansion in stimulating aggregate demand and supporting economic activity, as commonly discussed in economic growth literature.

7 CONCLUSIONS

The analysis demonstrates that the Bayesian BSQ model can effectively account for the heterogeneity of economic relationships observed in positively skewed data, such as the private consumption-to-GDP ratio. Compared with mean-based approaches, the quantile-based framework provides a more comprehensive representation of the data by allowing the effects of explanatory variables to vary across different regions of the conditional distribution.

These findings emphasize the importance of prior specification and loss function selection in Bayesian estimation. Although alternative prior specifications also demonstrate robustness and stability across most quantile levels, the Jeffreys prior generally provides more accurate and efficient estimates. Furthermore, the joint consideration of MAE and RMSE offers a more comprehensive evaluation of model performance, as these metrics measure average estimation error and sensitivity to large deviations, respectively. This result highlights the importance of using multiple evaluation criteria when assessing estimation accuracy.

The novelty of the present work lies in the Bayesian BSQ framework, which allows the incorporation of alternative prior distributions and loss functions while capturing heterogeneous effects across quantile levels. This provides a flexible modelling approach for analysing asymmetric economic relationships that cannot be fully characterized by conventional mean regression methods.

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