Development and Study of an Adaptive Stabilization Algorithm for a Self-Balancing Robot under Changing External Conditions

Yuri Zhigulevtsev[1], Shokhrukh Sultonov[2], Yorkinjon Valiev[2], Izzatilla Juraev[2] and Zilola Sultonova[2]
[1]*Department of Automatic Control Systems, Bauman Moscow State Technical University, 2nd Baumanskaya Str. 5, 105005 Moscow, Russia*
[2]*Department of Computer Science and Computer Graphics, Tashkent State Transport University, Temiryolchilar Str. 1, 100167 Tashkent, Uzbekistan*
specon@bmstu.ru, sultonovsh@yandex.ru, valiyev_y@tstu.uz, jorayev_i@tstu.uz, sultonovazr@gmail.com

Keywords: Two-Wheeled Self-Balancing Robot, Inverted Pendulum System, Optimal Control, PID Control, LQR Control, Dynamic Stabilization.

Abstract: Self-balancing mobile robots represent an important area of modern robotics, as they offer high maneuverability, compact size, and the ability to operate in confined spaces. Such systems are widely used in service robotics, transport systems, educational platforms, and autonomous mobile devices. The main challenge in the development of such robots lies in ensuring the stability of a dynamic system equivalent to an inverted pendulum, which is inherently unstable. This paper considers the development and study of a stabilization system for a two-wheeled self-balancing robot under changing external conditions, such as external disturbances and variations in platform mass. A control algorithm based on feedback from the tilt angle and angular velocity obtained from the MPU6050 inertial measurement unit is proposed. A complementary filter is used to process sensor data, providing fusion of gyroscope and accelerometer information and improving the accuracy of tilt angle estimation. To analyze the system dynamics, a mathematical model of the balancing robot was developed based on representing the system as an inverted pendulum on a cart. The model was transformed into state-space form, which made it possible to implement and investigate various control algorithms. Two types of controllers were implemented in this study: a classical PID controller and an optimal LQR controller. System simulation was carried out in the MATLAB/Simulink environment. Experimental studies included an analysis of the system response to various types of inputs, including impulse and step disturbances, as well as verification of stability under variations in robot mass. The obtained results showed that both control methods ensure system stability; however, the LQR controller demonstrates higher stabilization accuracy and smaller oscillations. The proposed stabilization algorithm can be used in the development of mobile robotic systems operating under uncertainty and changing external parameters.

1 INTRODUCTION

In recent years, there has been rapid development of mobile robotic systems capable of autonomous movement and performing various tasks under complex environmental conditions [1]. One of the promising directions in the development of mobile robots is two-wheeled self-balancing platforms, which are characterized by high maneuverability, compact design, and the ability to move in confined spaces [2]. Of particular interest are two-wheeled self-balancing robots whose dynamics are analogous to an inverted pendulum, a classical problem in automatic control theory [3]. Such systems are inherently unstable; therefore, their operation is possible only in the presence of an effective control system [2]. The study of inverted pendulum stabilization methods has a long history and is widely used as a benchmark problem for the development of new control algorithms [4].

Methods for stabilizing two-wheeled self-balancing robots using modern control theory approaches are being actively investigated [5]. For example, in the work of V.T. Nguyen et al. [6], an adaptive nonlinear proportional-derivative (ANPD) controller was proposed for a two-wheeled self-balancing robot modeled by the Lagrange equation with external forces. The simulation and experimental results showed that the TWSBR with the proposed ANPD controller quickly achieves balance and tracks target values with very small errors, demonstrating the efficiency and performance of the proposed controller. L. Guo et al. [7] proposed an optimal robot control method using reinforcement learning algorithms. The proposed optimal control scheme is capable of stabilizing the system and converging to the LQR solution obtained by solving the algebraic Riccati equation. P. Avramov et al. [8] developed a nonlinear electromechanical model of a two-wheeled self-balancing robot and proposed a cascade control algorithm based on sliding mode control. The proposed method provides stable robot stabilization and high robustness to external disturbances and parameter uncertainties. In [9], a robot design using sensor fusion methods for system state estimation was developed. Experimental results confirm the effectiveness of the proposed data fusion algorithm, showing that the robot maintains stability even when low-cost MEMS sensors are used. I. Jmel et al. [10] proposed a sliding mode control method with an adaptive observer, which enables the robot to maintain stability while moving on inclined surfaces and under external disturbances. G. Hadi [11] used optimization algorithms to tune the parameters of the LQR controller for a balancing robot. The author applies optimization methods, including the particle swarm optimization algorithm and the butterfly optimization algorithm, which makes it possible to improve stabilization quality and reduce settling time. R. B. Lakshmi et al. [12] proposed a combined control system integrating LQR and PID controllers. Such a hybrid approach provides faster settling time and less overshoot compared to using only one type of controller. In [13], a control method for a two-wheeled balancing robot based on the clonal selection algorithm was proposed, which is used to optimize the parameters of PID and LQR controllers. A. Abdelgawad et al. [14] carried out a comparative analysis of model-based and data-driven control methods for balancing robots. A systematic analysis of modern control methods is presented in the review paper by H. Zhang and N. M. Nor [15], which discusses various stabilization strategies for two-wheeled balancing robots, including PID controllers, LQR control, adaptive methods, and machine learning algorithms.

Despite the large number of studies, the problem of improving the stability of balancing robots under changing external conditions remains relevant. Variations in mass, external disturbances, and sensor inaccuracies can significantly affect system stability. The aim of this work is to develop and investigate a stabilization algorithm for a two-wheeled balancing robot using PID and LQR control methods, as well as to analyze system stability under changing external conditions.

2 MATHEMATICAL MODELING

2.1 Linear Model of the DC Motor

Two DC motors with gearboxes are used to drive the mobile robot. In developing the mathematical model, the dynamics of an individual motor are considered first, since they determine the relationship between the input voltage, armature current, angular velocity, and the generated torque. A simplified schematic of the DC motor model is shown in Figure 1.

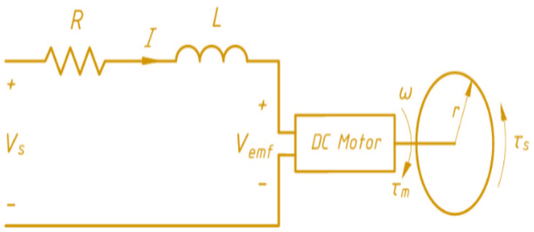


Figure 1: Simplified schematic of the DC motor model.

The electromagnetic torque of the motor is proportional to the armature current:

$$\tau_m = k_m i, \tag{1}$$

where k_m is the motor torque constant and i is the armature current.

When the shaft rotates in a magnetic field, a back electromotive force is generated, which is proportional to the angular velocity:

$$V_{emf} = k_e \omega, \tag{2}$$

where k_e is the back-EMF constant and ω is the angular velocity.

For the electrical circuit of the motor, Kirchhoff's law gives:

$$-V_s + iR + L\frac{di}{dt} + V_{emf} = 0, \tag{3}$$

where V_s is the supply voltage, R is the winding resistance, and L is the armature circuit inductance.

Substituting the expression for the back EMF, we obtain:

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{k_e}{L}\omega + \frac{V_s}{L}. \qquad (4)$$

The mechanical part of the motor is described by the rotational motion equation:

$$\tau_m - k_f\omega - \tau_s = I_R\dot{\omega}, \qquad (5)$$

where k_f is the viscous friction coefficient, τ_s is the load torque, and I_R is the armature moment of inertia.

After substituting $\tau_m = k_m i$, we obtain:

$$\dot{\omega} = \frac{k_m}{I_R}i - \frac{k_f}{I_R}\omega - \frac{1}{I_R}\tau_s, \qquad (6)$$

To simplify the analysis, it is assumed that the inductance L and the friction coefficient k_f are small and can be approximately taken as zero. Then:

$$i = \frac{k_e}{R}\omega + \frac{1}{R}V_s. \qquad (7)$$

Substituting this expression into the mechanical equation, we obtain the simplified dynamic model of the motor:

$$\dot{\omega} = \frac{k_m k_e}{I_R R}\omega + \frac{k_m}{I_R R}V_s - \frac{1}{I_R}\tau_s. \qquad (8)$$

For further control, system synthesis, it is convenient to write the model in state-space form. The shaft rotation angle θ and angular velocity ω are chosen as the state variables:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{k_m k_e}{I_R R} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \frac{k_m}{I_R R} & -\frac{1}{I_R} \end{bmatrix} \begin{bmatrix} V_s \\ \tau_s \end{bmatrix}. \qquad (9)$$

Thus, the obtained linear model makes it possible to evaluate the influence of the control voltage on the motor speed and position, and also to use it as part of the overall model of the self-balancing robot.

2.2 Dynamic Modeling of the Mobile Robot System

The two-wheeled self-balancing robot is considered as a dynamic system of the "inverted pendulum on a cart" type [16]. This model reflects the physical nature of the object: the robot body is located above the wheel axis, so even a small deviation from the vertical position leads to the occurrence of an overturning gravitational moment [17]. A simplified schematic of the robot is shown in Figure 2.

To derive the equations of motion, the wheels and the robot body are analyzed separately. Since the left and right wheels are symmetric, it is sufficient to consider the equations for one wheel and then account for the result for the wheel pair.

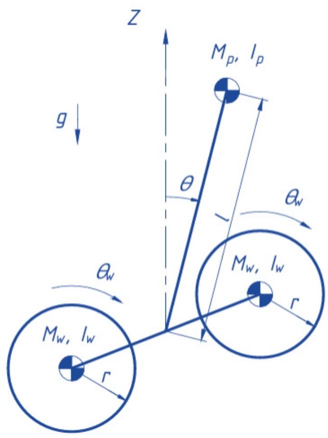


Figure 2: Simplified schematic of the robot.

According to Newton's second law for the right wheel:

$$F_{fr} - F_R = M_w\ddot{x}, \qquad (10)$$

where F_{fR} is the friction force, F_R is the reaction force of the body acting on the wheel, M_w is the wheel mass, and $\ddot{x}$ is the linear acceleration of the platform.

The moment equation with respect to the wheel center is:

$$C_R - F_{fr}r = I_w\ddot{\theta}_w, \qquad (11)$$

where C_R is the torque acting on the wheel, r is the wheel radius, I_w is the wheel moment of inertia, and θ_w is the wheel rotation angle.

Using the motor model, the wheel torque can be represented as:

$$C = -\frac{k_m k_e}{R}\dot{\theta}_w + \frac{k_m}{R}V_s. \qquad (12)$$

The transition from angular variables to linear motion is performed using the relations

$$\dot{\theta}_w = \frac{\dot{x}}{r}, \ddot{\theta}_w = \frac{\ddot{x}}{r}. \qquad (13)$$

Then, for two wheels, after summation, we obtain the first equation of motion of the robot:

$$2\left(M_w+\frac{I_w}{r^2}\right)\ddot{x}=-\frac{2k_mk_e}{Rr^2}\dot{x}+\frac{2k_m}{Rr}V_s-(F_L-F_R). \quad (14)$$

Next, the robot body of mass M_p, whose center of mass is located at a distance l from the wheel axis, is considered. Taking into account the reaction forces, gravity, and control torques, the translational and rotational motion equations of the body are formulated. After eliminating the intermediate forces, the system is reduced to the nonlinear form:

$$\begin{aligned}&\left(I_p+M_pl^2\right)\ddot{\theta}_p-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_s+\\&+M_pgl\sin\theta_p=-M_pl\ddot{x}\cos\theta_p,\end{aligned} \quad (15)$$

$$\begin{aligned}&\left(2M_w+\frac{2I_w}{r^2}+M_p\right)\ddot{x}+\frac{2k_mk_e}{Rr^2}\dot{x}+\\&+M_pl\ddot{\theta}_p\cos\theta_p-M_pl\dot{\theta}_p^2\sin\theta_p=\frac{2k_m}{Rr}V_s.\end{aligned} \quad (16)$$

These equations reflect the strong coupling between the longitudinal displacement of the platform and the body tilt angle. To stabilize the robot, it is necessary to move the wheels so as to compensate for the deviation of the center of mass from the vertical.

For controller synthesis, the system is linearized in the neighborhood of the upright equilibrium position. The substitution is introduced, where φ is the small deviation of the body from the vertical.

$$\theta_p=\pi-\varphi, \quad (17)$$

For small angles, the following approximations are assumed:

$$\cos\theta_p\approx-1,\sin\theta_p\approx-\varphi,\dot{\theta}_p^2\approx0. \quad (18)$$

Then the linearized equations of motion take the form:

$$\left(I_p+M_pl^2\right)\ddot{\varphi}-\frac{2k_mk_e}{Rr}\dot{x}+\frac{2k_m}{R}V_s-M_pgl\varphi=M_pl\ddot{x}, \quad (19)$$

$$\left(2M_w+\frac{2I_w}{r^2}+M_p\right)\ddot{x}+\frac{2k_mk_e}{Rr^2}\dot{x}-M_pl\ddot{\varphi}=\frac{2k_m}{Rr}V_s. \quad (20)$$

Based on these expressions, a state-space model is formed with the state vector

$$X=\begin{bmatrix}x & \dot{x} & \varphi & \dot{\varphi}\end{bmatrix}^T. \quad (21)$$

In standard form, the linear system is written as

$$\dot{X}=AX+BU, \quad (22)$$

where the elements of the matrices A and B are determined by the motor parameters, wheelbase, body, and center-of-mass position. To shorten the notation, the following definitions are introduced:

$$\alpha=I_p\beta+2M_pl^2(M_w+\frac{I_w}{r^2}). \quad (23)$$

$$\beta=2M_w+\frac{2I_w}{r^2}+M_p. \quad (24)$$

The obtained model forms the basis for stability analysis and the design of a closed-loop control system using PID and LQR controllers.

In the modeling process, the following assumptions are made: the wheels remain in continuous contact with the supporting surface, slipping is absent, and lateral forces during motion are neglected. These assumptions make it possible to focus on the longitudinal dynamics of the system and do not distort the main stabilization problem.

3 EXPERIMENTAL STUDIES

The experimental part of the work was aimed at verifying the operability of the developed model and assessing the control quality in a closed-loop system. The tests were carried out both in the MATLAB/Simulink environment and on a physical self-balancing robot platform (Fig. 3). The hardware part of the system included an Arduino UNO, an MPU6050 inertial module, motor drivers, and two DC wheel motors. To estimate the tilt angle, accelerometer and gyroscope data fused by a complementary filter were used. Based on the calculated angle, the microcontroller generated the control action and PWM signals for the motor drivers. In the study of the PID controller, simulations were performed with different types of input signals: impulse, step, and constant (Fig. 4).

The obtained results showed that under a short-term disturbance, the robot restores the upright position in less than 3 s even at a significant body deviation.

Under step input, the system maintains balance and ensures the movement of the platform to the specified position. In the constant angle reference mode, the robot is capable of maintaining the specified deviation, although a slight position offset is observed due to the physical coupling between tilt and motion.

To improve stabilization quality, an LQR controller synthesized on the basis of the linear state-space model was also implemented (Fig. 5). The optimality criterion was defined in the form of a quadratic cost functional:

$$J=\int_{0}^{\infty}\left(x^{T}Qx+u^{T}Ru\right)dt \tag{25}$$

where Q is the state weighting matrix and R is the control input weighting matrix.

According to the simulation results, the LQR controller provided a smoother transient response, smaller oscillations, and higher stability compared with the PID controller. The best results were obtained with an increased penalty on the tilt angle error, which made it possible to reduce the recovery time to approximately 2 s and decrease the deviation amplitude to ±2.3°. In addition, a series of tests was carried out with an increased platform mass.

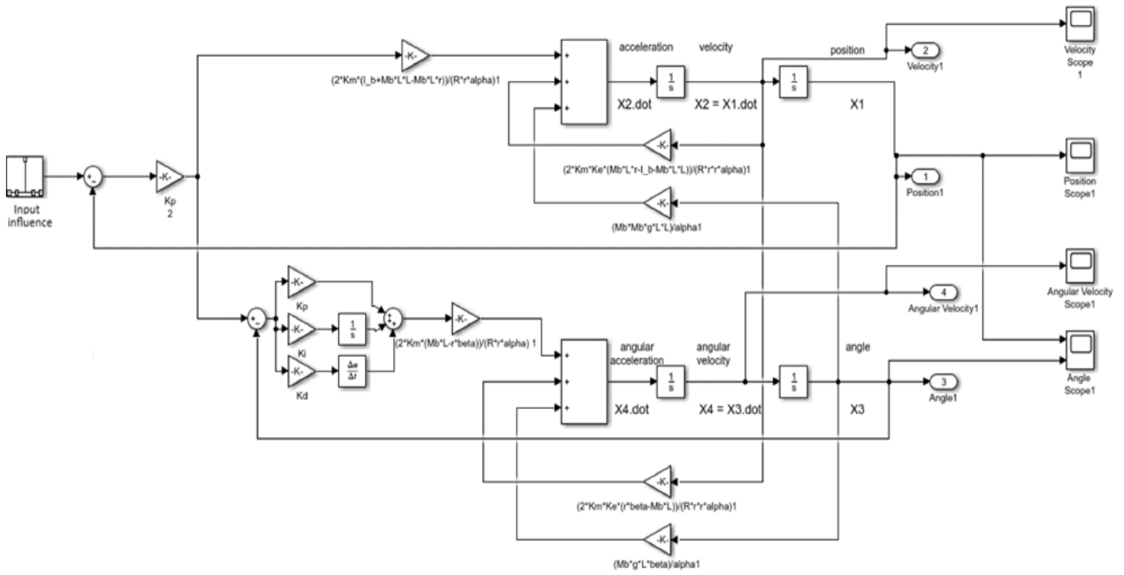
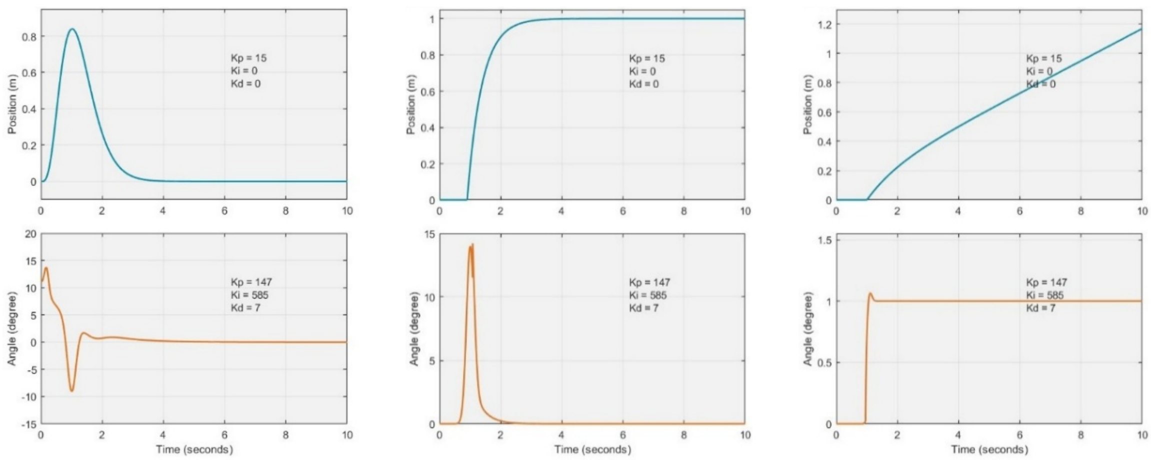


Figure 3: PID controller model for investigating robot behavior.



(a) (b) (c)

Figure 4: Position behavior and stabilization of the robot’s tilt angle under an impulsive disturbance: a), a step control input, b), and a prescribed tilt angle of 1° c).

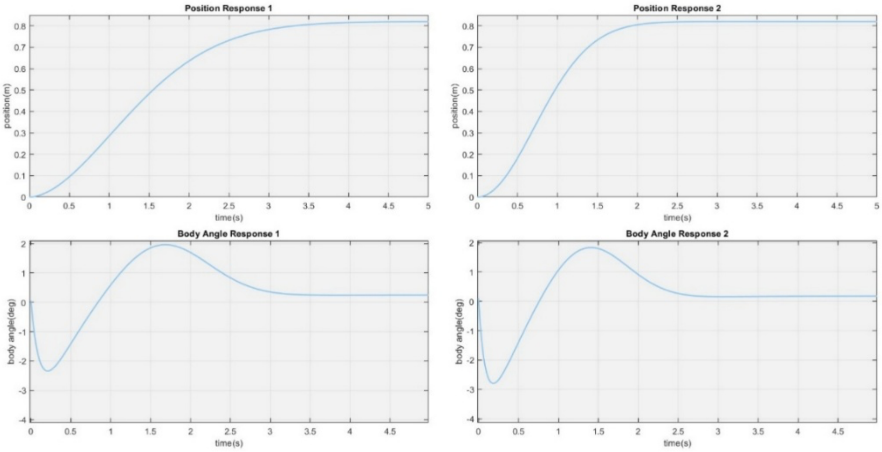


Figure 5: Simulation results with the LQR controller.

An additional load of approximately 500 g was mounted on the upper part of the robot. The experiment showed that the control system continued to operate without retuning the controller parameters. The robot retained its self-stabilization capability, while the increase in transient response time was only 0.3–0.5 s. This confirms the robustness and adaptability of the developed control system under changes in the mass and inertial characteristics of the object.

4 CONCLUSIONS

In this study, a stabilization system for a two-wheeled balancing robot was developed and investigated. A mathematical model of a two-wheeled self-balancing robot was constructed, including a linear DC motor model and a dynamic description of the robot as an inverted pendulum on a cart. Nonlinear equations of motion were derived, linearized, and converted into a state-space model suitable for control system synthesis.

The conducted studies showed that both PID and LQR controllers are capable of ensuring the stability of the robot in the upright position. At the same time, the PID controller is distinguished by its simplicity of implementation and convenience in practical application, while the LQR controller provides higher transient response quality, smaller oscillations, and more accurate stabilization. Experimental validation on a physical platform confirmed the adequacy of the developed model and the operability of the control system under real conditions, including external disturbance modes and changes in payload. A promising direction for further research is the development of adaptive control algorithms capable of automatically adjusting controller parameters depending on the current state of the system and external conditions. Also, of interest is the application of optimization methods for controller parameter tuning and the investigation of hybrid control systems combining the advantages of different approaches.

REFERENCES

[1] Q. Qian, J. Wu, and Z. Wang, "Dynamic Balance Control of Two-Wheeled Self-Balancing Pendulum Robot Based on Adaptive Machine Learning," International Journal of Wavelets, Multiresolution and Information Processing, vol. 18, no. 1, p. 1941002, 2020, [Online]. Available: https://doi.org/10.1142/S0219691319410029.

[2] A. Unluturk and O. Aydogdu, "Machine Learning Based Self-Balancing and Motion Control of the Underactuated Mobile Inverted Pendulum With Variable Load," IEEE Access, vol. 10, pp. 104706-104718, 2022, [Online]. Available: https://doi.org/10.1109/ACCESS.2022.3210540.

[3] J. A. Borja, I. Alvarado, and D. Muñoz De La Peña, "Low Cost Two-Wheels Self-Balancing Robot for Control Education Powered by Stepper Motors," IFAC-PapersOnLine, vol. 53, no. 2, pp. 17518-17523, 2020, [Online]. Available: https://doi.org/10.1016/j.ifacol.2020.12.2660.

[4] M. M. Sayyad, S. Nalawade, J. Palkar, et al., "Structural Analysis of Self-Balancing Robot," International Journal for Research in Applied Science and Engineering Technology, vol. 12, no. 1, 2024, [Online]. Available: https://doi.org/10.22214/ijraset.2024.58066.

[5] Q.-T. Do et al., "Modeling and Optimal Control for Two-Wheeled Self-Balancing Robot," Journal of Fuzzy Systems and Control, vol. 2, no. 1, pp. 22-28, 2024, [Online]. Available: https://doi.org/10.59247/jfsc.v2i1.162.

[6] V. T. Nguyen et al., "Adaptive Nonlinear PD Controller of Two-Wheeled Self-Balancing Robot with External Force," Computers, Materials and Continua, vol. 81, no. 2, pp. 2337-2356, 2024, [Online]. Available: https://doi.org/10.32604/cmc.2024.055412.

[7] L. Guo, S. A. A. Rizvi, and Z. Lin, "Optimal Control of a Two-Wheeled Self-Balancing Robot by Reinforcement Learning," International Journal of Robust and Nonlinear Control, vol. 31, no. 6, pp. 1885-1904, 2020, [Online]. Available: https://doi.org/10.1002/rnc.5058.

[8] Sh. Sultonov, J. Gulyamov, V. Bubnov, Sh. Shukurova, and N. Uralov, "Reliability of the Automatic Electric Heating System for Turnouts," in Lecture Notes in Networks and Systems, vol. 1547, Springer, 2025, pp. 11-17.

[9] J. Han, X. Li, and Q. Qin, "Design of Two-Wheeled Self-Balancing Robot Based on Sensor Fusion Algorithm," International Journal of Automation Technology, vol. 8, no. 2, pp. 216-221, 2014, [Online]. Available: https://doi.org/10.20965/ijat.2014.p0216.

[10] Jmel, H. Dimassi, S. Hadj-Said, and F. M'Sahli, "Adaptive Observer-Based Sliding Mode Control for a Two-Wheeled Self-Balancing Robot under Terrain Inclination and Disturbances," Mathematical Problems in Engineering, vol. 2021, no. 1, p. 8853441, 2021, [Online]. Available: https://doi.org/10.1155/2021/8853441.

[11] E. Nordlund, "Railway Turnout Heating Control and Fault Detection in Inductive Heating Systems," Master's thesis, Luleå University of Technology, Sweden, 2023.

[12] R. B. Lakshmi, S. M. Bashir, and G. S. J. Daniel, "Design, Simulation and Optimal Control of Two-Wheel Self-Balancing Robot Using LQR and PID," Tuijin Jishu/Journal of Propulsion Technology, vol. 45, no. 3, 2024.

[13] Y. Olmez, G. Ozmen Koca, and Z. H. Akpolat, "Clonal Selection Algorithm Based Control for Two-Wheeled Self-Balancing Mobile Robot," Simulation Modelling Practice and Theory, vol. 118, p. 102552, 2022, [Online]. Available: https://doi.org/10.1016/j.simpat.2022.102552.

[14] V. P. Bubnov, A. D. Khomonenko, and S. A. Sergeev, "Recursive Method for Generating the Coefficient Matrix of the System of Homogeneous Differential Equations Describing Nonstationary System Maintenance," in 2015 XVIII International Conference on Soft Computing and Measurements (SCM), St. Petersburg, Russia, May 19-21, 2015, id. 25, [Online]. Available: https://doi.org/10.1109/SCM.2015.7190415.

[15] L. Vázquez, *The Porous Medium Equation: Mathematical Theory*. Oxford, UK: Oxford University Press, 2006.

[16] R. A. D. D. Dharmasiri and M. K. Jayananda, "A Study of Two-Wheel Self-Balancing Robot Dynamics for Control Systems Learning," Vidyodaya Journal of Science, vol. 27, no. 2, 2024, [Online]. Available: https://doi.org/10.31357/vjs.v27i02.7834.

[17] M. S. Arani, H. E. Orimi, W.-F. Xie, and H. Hong, "Sliding Mode Control Design of a Two-Wheel Inverted Pendulum Robot: Simulation, Design and Experiments," in F. Janabi-Sharifi and W. Melek (eds.), *Advances in Motion Sensing and Control for Robotic Applications*, Lecture Notes in Mechanical Engineering, pp. 93-107, Springer, Cham, 2019, [Online]. Available: https://doi.org/10.1007/978-3-030-17369-2_7.