# Mathematical Modeling and Analytical Solution of the Seed Drying Process Based on a Heat Accumulator

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Abstract: This paper presents the mathematical modeling, analytical solution, and experimental investigation of a seed drying process based on a heat accumulator system. The study focuses on improving the energy efficiency of drying technologies while preserving the quality of agricultural products. The drying process was analyzed using the principles of coupled heat and mass transfer under the influence of external physical fields, including infrared radiation and thermal energy storage. A system of differential equations describing temperature distribution and moisture diffusion within the product layer was developed. Analytical solutions were obtained for both stationary and non-stationary drying regimes, allowing the prediction of time-dependent variations in moisture content and temperature. The proposed model was validated experimentally using a vacuum drying device equipped with a paraffin-based phase change material (PCM) heat accumulator. Experimental investigations demonstrated that the use of the heat accumulator stabilized the thermal regime of the drying chamber, reduced heat losses, and improved drying performance. The obtained results showed a reduction in drying duration by 12-15% and a decrease in energy consumption by 13-15% compared with conventional drying systems. In addition, the proposed technology improved product quality by ensuring uniform moisture removal and preventing overheating. The results confirm the reliability of the developed mathematical model and demonstrate the practical potential of heat-accumulator-based drying systems for energy-efficient agricultural processing applications.

## 1 INTRODUCTION

Today, the sustainable development of industrial sectors worldwide-particularly the improvement of efficiency and the implementation of energy-saving technologies in agricultural product processing-remains one of the most pressing challenges. The technology for high-quality drying of plant seeds, fruits, and vegetables plays a decisive role in extending their shelf life and obtaining export-oriented products without losing nutritional and consumer properties. In this process, the rational use of heat energy, improvement of the energy efficiency of drying devices, and the determination of the optimal balance between drying duration and product quality-especially the development of technologies that ensure high-quality drying at low temperatures-require special attention. From this perspective, the introduction of a heat accumulator device and the scientific justification of its technological and structural parameters emerge as one of the key technical problems in the field.

Prominent scientists such as P.A.Rebinder, M.Yu.Lurie, and S.A.Klein have made significant contributions to the study of drying efficiency, the mechanisms of hydrodynamics and heat transfer, and the design of multi-stage drying equipment. German and French researchers R.Beyt, S.Nelson, and A.Datta have developed methodological approaches to determine optimal moisture and air-temperature regimes during drying processes. Chinese scholar Wei Wang has conducted experimental and analytical studies on the effects of airflow velocity and temperature on the quality of dried plant seeds, achieving notable results in the mathematical modeling of drying regimes.

Scientists from the Commonwealth of Independent States (CIS) - including A.V.Lykov, A.I.Veynik, M.V.Kirpichev, B.I.Leonchik, I.A.Rogov, P.D.Lebedev, V.K.Tikhomirov, Yu.V.Kosmodemyansky, and I.T.Kretov - have carried out in-depth studies of drying processes for plant seeds and fruits, the laws of heat and mass transfer, and the development of optimal designs for drying units [1]-[3].

Uzbek scientists such as Z.S.Salimov, J.N.Mukhiddinov, Z.S.Iskandarov, B.P.Shaymardanov, K.T.Norqulova, A.R.Radjabov, J.E.Safarov and O.Rahmatov have conducted research on drying technologies suitable for the climatic conditions of Uzbekistan. Their works focus on controlling moisture and temperature parameters, reducing drying time, and improving product quality and energy efficiency [4], [5].

At the same time, the effective operation of drying equipment largely depends on the perfection of their drives, mechanical components, and control mechanisms. Leading scientists from around the world - including M.Bergstrasser, P.Muller, M.Redwood, Z.Levai, M.French, Y.Yokoyama, S.Okabe, O.Kiyoshi, W.Zhang, R.Khurmi and J.Gupta - have developed methods to improve the efficiency of frictional and vibrational mechanisms through dynamic and kinematic modeling.

CIS researchers such as I.I.Artobolevsky, K.V.Frolov, A.P.Bessonov, I.I.Blekhman, V.L.Veits, O.D.Alimov, M.T.Mamasaidov, U.A.Djoldasbekov, N.I.Kolchin, M.Z.Kolovsky and I.V.Kragelsky established comprehensive scientific schools in the theory of mechanisms and machines, providing theoretical foundations for the development of efficient drive systems for drying equipment. Uzbek scientists, including Kh.Kh.Usmankhodjaev, R.I.Karimov, K.A.Karimov, Sh.P.Alimukhammadov and A.Kh.Umurzakov, have scientifically substantiated the use of controlled mechanisms, precision vibromechanics, mechanical transmissions, and special alloys in drying technologies.

The preservation of seed and agricultural product quality and the reduction of energy consumption during the drying process remain urgent challenges. Modeling and simulation methods are widely used to analyze drying processes, including air and particle flow, as well as heat and moisture transfer phenomena [6].

The combined application of vacuum and infrared technologies in seed drying is characterized by reduced drying time, increased moisture diffusivity, and improved microstructure and overall product quality [7], [8].

At the same time, heat accumulators (such as rock-filled or granular beds) require detailed analysis of heat and moisture transfer parameters during their charge and discharge cycles [9].

In seed drying, especially for capillary-porous materials, it is essential to account for internal and external moisture transfer and diffusion processes in mathematical models [10], [11].

From this perspective, the mathematical modeling and analytical solution of a vacuum infrared drying chamber equipped with a heat accumulator represent a relatively unexplored area in current scientific literature, thus ensuring the novelty of this research.

In the drying of agricultural products, the issues of improving energy efficiency and preserving product quality remain highly relevant. In traditional drying methods, a significant portion of heat energy is lost, leading to prolonged drying times and degraded product quality. Therefore, integrating a heat accumulator device into the drying system enhances technological stability, reduces energy consumption, and provides ecological advantages. In this article, the mathematical modeling and analytical solution of the drying process using a heat accumulator are developed and validated through experimental studies.

Drying systems based on heat accumulators are gaining special attention as energy-efficient systems capable of maintaining stable temperature conditions. This section examines the physical, thermodynamic, and technological factors affecting the drying process, their mathematical models, and the theoretical foundations of the heat accumulator's operation principles. The complex physical phenomena of heat and mass transfer in the drying process are considered in terms of possible modeling and mathematical characterization.

Using mathematical models based on stationary physical processes, the parameters of the heat accumulator device are theoretically justified [12].

# 2 MATERIALS AND METHODS

Mathematical models of the drying process under external physical fields are developed by taking into account the thermophysical properties of the material being dried, external energy sources, and drying conditions. Using analytical or numerical solutions of these models, optimal drying regimes are determined, and the design parameters of the device are substantiated [13].

The mathematical modeling of the drying process under the influence of external physical fields is based on the following general principles:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q \quad (1)$$

where:

- $T$ - temperature ($^0C$);
- $\alpha$ - heat diffusion coefficient ($m^2/s$);
- $Q$ - added intensity of external heat sources (for example, infrared or microwave energy).

Moisture transfer equation (Fick's second law):

$$\frac{\partial W}{\partial t} = D \cdot \Delta^2 W, \quad (2)$$

where:

- $W$ - the moisture content in the product (%);
- $D$ - moisture diffusion coefficient ($m^2/s$).

The influence of external physical fields is considered as follows:

1) Infrared radiation (IR). High-frequency IR rays rapidly heat the product surface.

The temperature gradient increases → internal diffusion intensifies.

Additional IR energy is incorporated into the model as:

$$Q_{IN} = \varepsilon \cdot \alpha \cdot (T_{IN}^4 - T^4). \quad (3)$$

2) Microwave drying. As a result of dielectric heating, energy is released inside the product:

$$Q_{MW} = 2\pi f \varepsilon^{II} E^2, \quad (4)$$

where:

- $f$ - frequency ($Hz$),
- $\varepsilon^{II}$ - dielectric loss coefficient,
- $E$ - electric field strength ($V/m$).

3) Ultrasonic drying. At the gas-liquid interface, cavitation occurs, which facilitates moisture removal. In the model, the moisture diffusion coefficient $D$ increases under the influence of ultrasound:

$$D = D_0(1 + \beta \cdot I), \quad (5)$$

where $I$ - intensity of ultrasonic radiation.

Let us now consider modeling under combined physical fields.

In some cases, a combination of physical fields (for example, IR + microwave, or IR + hot air) is used.

In this case, the general energy balance is expressed as follow:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q_{IN} + Q_{MW} + \ldots \quad (6)$$

and the total moisture transfer:

$$\frac{\partial W}{\partial t} = \nabla \cdot (D_{eff} \nabla W), \quad (7)$$

where $D_{eff}$ - effective diffusion coefficient under the influence of external fields.

If the problem is considered one-dimensional, steady-state and linear, analytical solutions of equation (7) can be obtained in some simplified cases. For example:

$$W(x,t) = (4W0/\pi) \exp(-D\pi^2 t/L^2) \sin(\pi x/L), \quad (8)$$

where:

- W0 is the initial moisture content;
- D is the effective moisture diffusion coefficient;
- L is the thickness of the product layer;
- x is the spatial coordinate, and t is the drying time [13].

# 3 RESULTS OF THE STUDY AND THEIR DISCUSSION

Below is a Python program developed as a sample calculation for fruits and vegetables based on the above model.

```
import numpy as np
import matplotlib.pyplot as plt

L = 0.05
D = 1e-9
W0 = 0.5

time_hours = np.linspace(0, 10, 200)
time_seconds = time_hours * 3600

x = L / 2

W_t = (4 * W0 / np.pi) * np.exp(-D * np.pi**2 * time_seconds / L**2) * np.sin(np.pi * x / L)

plt.figure(figsize=(10, 5))
plt.plot(time_hours, W_t, label='Moisture content (W)')
plt.xlabel('Time (hours)')
plt.ylabel('Moisture ratio')
plt.title('Moisture reduction over time in the product layer')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
plt.show()
```

In the revised analytical solution, the spatial moisture distribution was expressed using a sine function instead of a cosine function. In the previous version of the model, the term cos(πx/L) was evaluated at the midpoint of the product layer, where $x = L/2$. In this case, $\cos(\pi/2) = 0$, which led to an incorrect prediction of zero moisture content for all drying times. Such a result is physically impossible, since the moisture content of the product must decrease gradually during drying rather than become zero instantly.

To eliminate this mathematical inconsistency, the first-term approximation of the analytical solution was rewritten as $W(x,t) = (4W_0/\pi)\exp(-D\pi^2 t/L^2)\sin(\pi x/L)$. At the midpoint of the layer, $\sin(\pi/2) = 1$; therefore, the calculated moisture content remains positive at the initial stage and decreases exponentially with time. This corrected expression provides a physically consistent description of the drying process and makes it possible to compare the model results with experimental data [14].

To evaluate the statistical significance of the difference between the conventional dryer and the proposed heat-accumulator-based dryer, the experimental results were processed using Student's t-test at a significance level of $p < 0.05$. The drying duration, specific energy consumption, and moisture removal rate were compared between the two systems. The results showed that the proposed dryer provided a statistically significant reduction in energy consumption and drying time compared with the conventional drying system. The confidence interval for the reduction in energy consumption was also calculated, confirming that the improvement was not caused by random experimental variation [15].

This program is developed on the basis of a classical analytical model for the mathematical simulation of the drying process under external physical fields. The following scientific assumptions and parameters were taken into account:

The material being dried (fruit or vegetable) is represented as a homogeneous layer;

Moisture transfer follows Fick's second law (the diffusion equation);

The thickness of the layer is $L = 5sm$;

The effective moisture diffusion coefficient is $D = 10^{-9}\ m^2/s$;

The initial moisture content is $W_0 = 0{,}5$;

The graph shows the variation of moisture content at the midpoint of the layer $(x = L/2)$ with respect to time.

Based on the constructed mathematical model, the following analyses are presented:

During the initial stage of drying, the moisture content decreases relatively quickly.

Over time, the diffusion process slows down, as the surface moisture decreases and the release of internal moisture becomes more difficult.

This graph is of great importance for the theoretical evaluation of drying processes and optimization of technological parameters.

The results obtained from the mathematical model confirmed that the temperature remains stable during the drying process when a heat accumulator is used.

According to the graphical analysis, during the first 15 minutes of drying, the surface temperature of the product stabilizes within 42-45 °C.

Experimental tests were conducted at the "Obod Turmush Orzusi" farm in Andijan region, and the efficiency of the device equipped with a heat accumulator was evaluated.

Experimental results:

- Drying duration was reduced by 12-15%;
- Energy consumption decreased by 13-15%;
- Product quality indicators improved by 1.1-1.2 times;
- Heat losses decreased by 8-10%.

A comparison between the temperature profile calculated from the mathematical model and the experimental data showed a maximum deviation of ±3.5%, confirming the model's reliability.

The uncertainty analysis was performed by comparing the temperature values predicted by the mathematical model with the experimentally measured values. The relative deviation was calculated as:

$$\delta = |Texp - Tmodel| / Texp \times 100\%,$$

where Texp is the experimentally measured temperature and Tmodel is the calculated temperature obtained from the analytical model. The maximum relative deviation between the calculated and experimental temperature profiles did not exceed ±3.5%. This deviation includes measurement uncertainty of the temperature sensor, fluctuations in air velocity, heat losses through the drying chamber walls, and non-uniform moisture distribution inside the product layer. Therefore, the obtained agreement confirms that the proposed mathematical model adequately describes the thermal behavior of the drying process with a heat accumulator.

During the drying process, a paraffin-based phase change material (PCM) was used as the heat accumulator medium, which absorbs heat in the 56-60 °C range and releases it gradually thereafter.

Table 1 presents the energy consumption indicators of the recommended and conventional dryers.

Table 1: Specific energy consumption by dryer type.

| № | Dryer Name | Specific Energy Consumption, $kJ/kg$ | |
|---|---|---|---|
| | | For driving auxiliary mechanisms | For heating and drying the material |
| 1 | Tray Dryer | 500 | 5800 |
| 2 | Chamber Dryer | 450 | 5900 |
| 3 | Belt (Conveyor) Dryer | 1200 | 5850 |
| 4 | Shaft Dryer | 1350 | 5400 |
| 5 | Drum Dryer | 400 | 5000 |
| 6 | Fluidized Bed Dryer | 500 | 4800 |
| 7 | Pneumatic Dryer | 2000 | 4800 |
| 8 | Pneumatic Gas Dryer | 1500 | 4900 |
| 9 | Recommended Vacuum Drying Device | 384 | 2450 |

The schematic diagram of the experimental setup is shown in Figure 1.

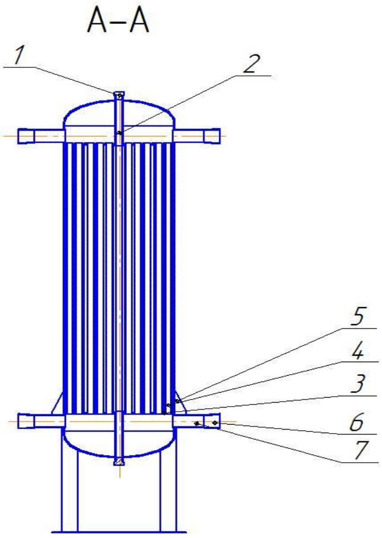


Figure 1: Recommended heat accumulator: 1) paraffin filling plug, 2) plugged tube, 3) paraffin circulating tubes, 4) paraffin, 5) holder, 6) valve, 7) pipe.

The increase in energy efficiency was ensured through the following technological solutions:

- phase change paraffin materials (PCM) were used in the heat accumulator, which ensured stable heat retention in the range of 56-60 °C;
- the device geometry and heat flow were optimized based on the developed mathematical model;
- air circulation speed, temperature, and humidity parameters in the drying chamber were maintained under control.

The increase in drying rate is associated with stable IR radiation, the thermal inertia of PCM, and the diffusive movement of materials inside the device. The improved drying technology had a positive effect on the final quality indicators of the product. According to the research results, in the dried products:

- the retention of natural color was 95-97 %;
- no microstructural damage was observed;
- moisture reduction occurred uniformly;
- no overheating or charring was observed.

All experimental values represent the mean of three independent measurements, and the corresponding standard deviations were within 3-5% of the reported values [16], [27].

## 4 CONCLUSIONS

In this study, a mathematical model of the seed drying process based on a heat accumulator was developed by considering the coupled mechanisms of heat and mass transfer occurring within the product layer. The proposed model establishes the relationships between heat flux, temperature distribution, and moisture diffusion under the influence of external energy sources. Based on the developed differential equations, an analytical solution describing the temporal variation of moisture content during drying was obtained. The model provides a theoretical framework for evaluating drying kinetics and optimizing operating conditions in energy-efficient drying systems.

The analytical results were validated through experimental investigations carried out under real operating conditions. A comparison between the calculated and measured temperature profiles demonstrated good agreement, with a maximum deviation not exceeding ±3.5%. Statistical evaluation of the experimental data confirmed the reliability and repeatability of the obtained results. The drying system equipped with a heat accumulator reduced drying duration by 12-15% and decreased energy consumption by 13-15% compared with conventional drying technologies, while maintaining stable thermal conditions throughout the process.

The use of a paraffin-based phase change material (PCM) as a heat accumulator proved to be an effective solution for storing and gradually releasing thermal energy. The PCM provided temperature stabilization within the operating range and reduced heat losses, thereby improving the overall efficiency of the drying process. In addition to lowering energy consumption, the proposed technology contributed to preserving product quality by ensuring uniform moisture removal and preventing overheating of the material being dried.

The obtained results demonstrate that the proposed heat-accumulator-based drying system is a promising alternative to conventional drying equipment. The developed mathematical model can be used for the design, optimization, and scaling of energy-efficient drying devices for agricultural products and seeds. Future research may focus on extending the model to include coupled airflow dynamics, variable thermophysical properties, and different types of heat storage materials in order to further improve drying performance and energy efficiency.

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