

# Probabilistic Inventory Optimization Using Topp-Leone Distribution

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**Abstract:** This study aims to optimize probabilistic inventory management by employing two distributions from the Topp–Leone family, namely the Topp–Leone–Shanker (TL–Sh) and the Topp–Leone–Rayleigh (TL–RR) distributions. The research focuses on estimating key indicators of the probabilistic Economic Order Quantity (EOQ) model, including the optimal order quantity, reorder point, and various inventory-related costs such as holding, shortage, ordering, and total expected costs. The empirical application is based on real production data from the Diyala State Company, represented by demand quantities for electrical transformers (11/250 KV) over the period 2019–2021. The parameters of the proposed distributions were estimated using the Maximum Product of Spacings (MPS) method. Goodness-of-fit criteria (AIC, AICc, and BIC) and statistical tests (Anderson–Darling, Cramér–von Mises, and Kolmogorov–Smirnov) confirmed that the TL–Sh distribution provides a better fit to the observed demand data compared to the TL–RR and baseline Shanker distributions. Based on these results, inventory model indicators were computed for both distributions. The findings reveal that the probabilistic inventory model based on the TL–Sh distribution yields significantly lower total expected costs compared to the TL–RR model, indicating its superior performance in capturing demand uncertainty and optimizing inventory decisions. The study highlights the importance of selecting appropriate probability distributions in stochastic inventory models and demonstrates the effectiveness of the TL–Sh distribution in improving inventory control and reducing overall costs.

## 1 INTRODUCTION

Inventory is considered one of the most fundamental functions in any organization, whether commercial, industrial, or agricultural. The purpose of maintaining inventory lies in establishing principles and rules that enable organizations to reduce storage-related costs, meet demand requirements, and achieve balance. The core objective of inventory management is to enhance customer service levels, avoid shortages, and minimize the total inventory cost. To effectively control the inventory system, it is essential to determine both the reorder point and the optimal order quantity. Efficient control also requires avoiding excessive stock (overstock), which leads to unnecessary storage costs, as well as understock situations that result in losses due to the inability to meet market demand. In practice, inventory serves a critical role in providing protection against short-term fluctuations, and thus maintaining sufficient stock in production facilities helps mitigate unexpected variations. Accordingly, obtaining accurate estimates

of inventory indicators becomes a vital necessity to prevent losses caused by poor inventory management. With the rapid technological developments in production and marketing processes, inventory management has become increasingly complex. Demand patterns for replenishment are now characterized by randomness, which necessitates the use of suitable probability distributions to model such data. Employing appropriate distributions allows for more precise estimation of inventory indicators and contributes to minimizing the total inventory cost. Against this background, the present study focuses on estimating the probabilistic inventory model indicators based on real-world data regarding demand for electric transformers. Specifically, the study applies the Topp–Leone–Shanker (TL–Sh) and Topp–Leone–Rayleigh Rayleigh (TL–RR) distributions and compares the resulting indicators to determine the more efficient distribution with respect to inventory costs.

Several studies have addressed inventory indicators. For instance, [1] Al-Sabawi and Hussein (2013) analyzed a multi-period probabilistic dynamic

inventory model for a single item across N periods. Their study estimated the minimum expected total cost, the optimal order size, and the ideal replenishment periods. The analytical treatment relied on probabilistic dynamic programming, and the results indicated optimal replenishment periods at (1, 10, 28), with an optimal order quantity of 34,974 units, and a minimum expected total cost of 888.487. Likewise, [2] in 2014, Salman investigated the importance of using probabilistic and non-probabilistic inventory models to support blood banks in determining the optimal stock level of blood units by type. The study also estimated the optimal inventory level that minimizes the total cost, defined in terms of maximizing patient treatment and minimizing waiting times. The model employed geometric programming to simplify the blood bank inventory problem, thereby reducing costs while maintaining optimal stock and service levels. The findings confirmed the effectiveness of geometric programming in lowering total cost, reducing waiting times, and achieving the economic order quantity (optimal inventory size).

## 2 INVENTORY CONCEPT

Storage is regarded as one of the most essential functions within companies and institutions, as it involves organizing and managing the processes of storing goods and materials, preserving them, and ensuring their timely provision to the intended recipients. In practice, demand quantities for goods vary depending on consumer requirements for different products. Similarly, waiting times are not fixed, as they are influenced by external supply conditions, which may at times delay the arrival of orders. Therefore, it is necessary to achieve a balance between the inventory levels required to meet consumer demand and the associated costs. Excess inventory creates additional challenges, as it results in immobilized capital and unnecessary utilization of storage space, while insufficient inventory leads to losses due to the inability to satisfy actual demand. Consequently, firms face the critical challenge of determining the optimal inventory level, the appropriate timing for issuing purchase orders, and the economic order quantity for each replenishment cycle. Inventory can thus be defined as the stock of materials or goods retained for future use or consumption when needed. Inventory level refers to the quantity of goods available in storage at a given time. This level fluctuates over time- declining as consumption increases and rising upon the

replenishment of goods with the arrival of new orders [3] - [5].

### 2.1 Inventory Costs

The main categories of inventory- related costs can be summarized as follows [6]-[8]:

- **Ordering Cost.** This represents the fixed cost incurred for each order once it is processed. It includes expenses such as transportation, inspection of defective items, insurance, administrative costs of preparing specifications and purchase orders, negotiations with suppliers, opening letters of credit, inspection, and potential damage to stored goods. As the order quantity increases, this cost decreases; however, frequent reordering leads to higher total ordering costs.
- **Holding (Carrying) Cost.** This cost encompasses all expenses associated with holding inventory, including warehouse rent or depreciation, operating expenses, taxes, equipment rental and depreciation, labor costs, borrowing costs, and insurance premiums on inventory.
- **Purchasing/Production Cost.** This cost is generally fixed but may decrease when producing or purchasing in larger quantities, as discounts are often granted for bulk orders that exceed a certain threshold.
- **Shortage (Stockout) Cost.** This arises when demand cannot be met due to insufficient inventory. It includes costs such as lost sales, production delays, and increased cost of raw material.

### 2.2 Fundamental Inventory Concepts

The fundamental concepts of inventory management form the basis for effective control and decision-making within inventory systems:

- **Demand.** Demand for an item is a key factor directly influencing the order size across different time periods. Demand can generally be classified into two types: deterministic demand and probabilistic demand [9], [10].
- **Reorder Point.** The reorder point occurs when inventory needs to be replenished, based on projected future demand, past consumption patterns, and forecasts. Its purpose is to ensure product availability without causing excessive stock or shortages. The determination of the reorder point depends on several factors, including the inventory management policy,

supplier lead time, and rate of consumption. In other words, it represents the stock level that should cover consumption during the replenishment lead time. While setting the reorder point, daily usage, lead time for the economic order quantity, and safety stock must be considered. The lead time should be sufficient to complete procurement and avoid delays that may affect customers [11], [12].

- Safety Stock. Safety stock, or buffer inventory, refers to the reserve quantity held by a company at all times regardless of the current inventory level. It serves as a safeguard against uncertainties and unexpected fluctuations in demand or supply [13].
- Lead Time. Lead time is the time interval between issuing a purchase order and receiving it, in the case of procurement from external suppliers. If the items are produced internally, lead time refers to the time required to manufacture and receive the production lot [14].
- Maximum Inventory Level. This is the highest allowable stock level of a given item. Setting a maximum inventory limit is intended to avoid negative consequences such as product damage or spoilage, increased storage costs, and capital being tied up unnecessarily in inventory [4].

### 2.3 Inventory Models

Several models have been developed to mathematically represent inventory systems. Among the most widely used is the Economic Order Quantity (EOQ) model, which exists in two forms: the classical (deterministic) EOQ model and the probabilistic EOQ model. These are outlined as follows:

#### 2.3.1 Classical EOQ Model

The Economic Order Quantity (EOQ) can be defined as an inventory management system that determines the optimal order size of an item to minimize the total cost, which includes both ordering and holding costs. The model aims to strike a balance between the cost of placing orders and the cost of carrying inventory. It provides organizations with the optimal EOQ value to be ordered each time in order to achieve the minimum possible total cost of procurement and storage. The EOQ model was first introduced by Ford W. Harris in 1913 and has since become one of the most commonly applied techniques in inventory control due to its relative simplicity and practical

usefulness. The model is based on several assumptions [15]:

- 1) Demand for a specific item is known, constant, and independent of other items.
- 2) Lead time (the time between placing and receiving an order) is known and constant.
- 3) Inventory replenishment is instantaneous, i.e., orders are received in full and at once.
- 4) Quantity discounts are not allowed.
- 5) Stockouts are completely avoidable.

The classical EOQ model is mathematically represented by the following equation:

$$(EOQ)Q^* = \sqrt{\frac{2DK}{h}},$$

$$Total\ cost = \left(\frac{D}{Q}\right) * K + \left(\frac{Q}{2}\right) * h + D * A,$$

where:

- $EOQ(Q^*)$ : Order quantity;
- $D$ : Expected demand per unit time;
- $h$ : holding cost per inventory unit per unit time;
- $K$ : Setup cost per order;
- $A$ : Purchasing Cost.

#### 2.3.2 Cost Calculations

$$Annual\ Ordering\ Cost = \left(\frac{D}{Q}\right) * K.$$

$$Annual\ Holding\ Cost = \left(\frac{Q}{2}\right) * h.$$

$$Annual\ Purchasing\ Cost = D * A.$$

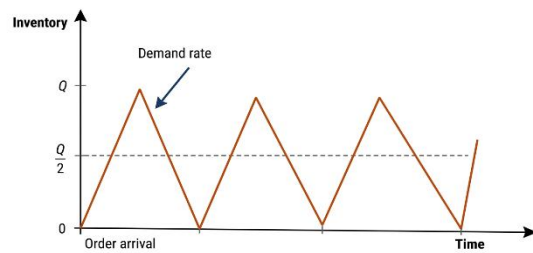


Figure 1: Economic Order Quantity (EOQ)<sup>1</sup>.

In the Figure 1 illustrates the relationship between inventory level and the time it remains in storage. The inventory level starts from zero, representing the minimum stock, and increases upon the arrival of an order quantity, denoted by  $Q$ , which corresponds to the maximum inventory level. Over time, the inventory is gradually consumed until it is completely depleted, repeating this cycle with each subsequent order.

### 2.3.3 Probabilistic EOQ Models

These models represent cases in which demand is probabilistic or random in nature and are analyzed based on the principles of probability theory by assigning an appropriate probability distribution to describe it. The primary objective of such models is to determine the expected value and standard deviation of the random variable(s), or to minimize the expected total cost of the inventory problem under consideration. These models are generally classified into two categories [16].

- Stationary models. Characterized by a stable probability distribution of demand over a given time horizon.
- Non-stationary models. In contrast, this type is characterized by an unstable probability distribution of demand over a given time horizon.

The probabilistic EOQ model is based on the following assumptions:

- 1) Unfilled demand during lead time is backlogged.
- 2) No more than one outstanding order is allowed.
- 3) The distribution of demand during the lead time remains stationary with time.

#### 2.3.3.1 Mathematical Formulation of the Probabilistic Inventory Model

In this model, demand is a random variable that follows a known probability distribution. The primary objective of the probabilistic model is to determine  $(y,R)(y, R)(y,R)$  that minimize the total cost, which consists of (ordering cost, holding cost, and shortage cost) per unit of time (Fig. 2). Let  $x$  denote the demand during the lead time, which is a random variable with probability density function  $f(x)$ . The notations of the probabilistic EOQ model are defined as follows:

- $D$ : Expected demand per unit time;
- $h$ : Holding cost per inventory unit per unit time;
- $K$ : setup cost per order;
- $p$ : shortage cost per inventory unit;
- $R$ : Reorder Point;
- $y$ : order quantity per inventory cycle;
- $S$ : expected shortage per inventory cycle;
- $A$ : cost of a single order;
- $F(x)$ =pdf of demand,  $x$ , during lead time.

#### 2.3.3.2 Specification of the Cost Function Components

First: Ordering Cost (Setup Cost):

$$\text{Setup Cost} = \frac{KD}{y} \tag{1}$$

Second: Expected Holding Cost.

Let  $I$  denote the average inventory level. Then, the expected holding cost per unit of time is given by  $(hI)$ .

The expected value of the inventory units held during the inventory cycle is equal to:

$$I = \frac{(y+E\{R-X\})+E\{R-X\}}{2}$$

The expected holding cost per unit of time is given by:

$$I = \frac{y}{2} + R - E\{X\} \tag{2}$$

Thus, the holding cost is equal to:

$$\text{holding cost} = h \left( \frac{y}{2} + R - E\{X\} \right) \tag{3}$$

The expected number of units during the inventory cycle equals (the expected inventory level at the beginning of the cycle + the expected inventory level at the end of the cycle)/2.

The expected inventory level at the beginning of the cycle =  $y + E\{R - X\}$ .

The expected inventory level at the end of the cycle =  $E\{R - X\}$ .

In the case where  $E\{R - X\}$  is negative, the expression is disregarded.

Expected Shortage Cost. This occurs when the cost value  $(X > R)$ , also referred to as the expected shortage quantity in a single cycle. Its expected value is calculated for each cycle as follows:

$$S = \int_R^\infty (X - R) * f(x) dx, \tag{4}$$

$$\begin{aligned} \bar{S} &= E[S(X)] = \int_0^\infty S(X) f(X) dx \\ &= \int_0^R 0 f(x) dx \\ &\quad + \int_R^\infty (X - R) f(x) dx \\ \bar{S} &= \int_R^\infty (X - R) f(x) dx \end{aligned} \tag{5}$$

Let denote the expected number of shortage units during the inventory cycle:

For  $X \leq R$ , we have  $S(X) = 0$ , and for  $X > R$ , we have  $S(X) = X - R$ .

Since the shortage cost  $p$  is proportional only to the shortage quantity, the expected shortage cost per cycle is  $p\bar{S}$ . Moreover, as the number of cycles per unit time is  $\frac{D}{y}$ , then:

$$\text{Shortage cost} = p\bar{S} \frac{D}{y}. \quad (6)$$

In terms of the two variables  $(y, R)$ , the expected total cost per unit time can be expressed, denoted by  $TAC(Y, R)$ :

$$TAC(y, R) = K \frac{D}{y} + h \left( \frac{y}{2} + R - E(X) \right) + p\bar{S} \frac{D}{y}, \quad (7)$$

$$y^* = \sqrt{\frac{2D(K + p\bar{S})}{h}}, \quad (8)$$

$$\int_R^\infty f(x) dx = \frac{hy^*}{pD}. \quad (9)$$

By substituting (5) into (8), we obtain:

$$\hat{Y} = \sqrt{\frac{2D(K + PE(x))}{h}}. \quad (10)$$

We also obtain:

$$\tilde{Y} = \frac{PD}{h}. \quad (11)$$

From (9) and (10), we observe the difficulty of finding the optimal values for  $(y^*, R)$ . Therefore, we will use the numerical algorithm whose steps are outlined below:

Step 1: Obtain an initial solution by assuming that:

If  $R_0 = 0$  then  $y_1 = y^* = \sqrt{\frac{2KD}{h}}$ , Then we proceed to the next step.

Step 2: Use the value of  $y_1$  to find  $R_1$  by employing (10) and then determine the value of  $y_2$ .

Step 3: Compute the value of  $R_2$  based on  $Y_2$  using (10). If  $R_2 \approx R_1$ , we stop the procedure, and the optimal solution is  $Y^* = Y_2$  and  $R^* = R_2$ . Otherwise, repeat the above step to find successive values  $R_i$  and  $Y_i$  until the optimal solution is obtained.

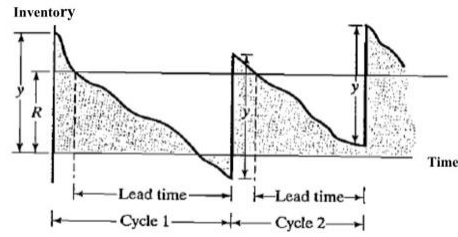


Figure 2: Probabilistic inventory model with shortage<sup>2</sup>.

## 2.4 Probability Distributions for Demand Quantities

The mathematical formula for the Topp-leone family is known by the (12), (13) [17]:

$$f_{TL}(x) = 2\alpha g(x)\bar{G}(x) \left[ 1 - (\bar{G}(x))^2 \right]^{\alpha-1}; \alpha > 0 \quad (12)$$

$$F(x, \alpha) = [1 - (\bar{G}(x))^2]^\alpha; x > 0, \alpha > 0 \quad (13)$$

The probability density and distribution functions for the Shanker distribution are also defined by the two (14), (15);

$$G(x, \theta) = 1 - \left[ \frac{(\theta^2 + 1) + \theta x}{\theta^2 + 1} \right] e^{-\theta x}; x > 0, \theta > 0 \quad (14)$$

$$g(x, \theta) = \frac{\theta^2}{\theta^2 + 1} (\theta + x) e^{-\theta x}; x > 0, \theta > 0 \quad (15)$$

By substituting (14) and (15) into (12), we obtain the probability density function of the Topp-Leone-Shanker distribution, as shown in (16).

### 2.4.1 Topp-Leone-Shanker Distribution

The random variable  $X$  follows the Topp-Leone-Shanker distribution, and its probability density function (PDF) is represented by the following equation [18]:

$$g_{TL-sh}(x; \alpha, \theta) = 2\alpha \left[ \frac{\theta^2}{\theta^2 + 1} (\theta + x) e^{-\theta x} \right] \left[ \frac{(\theta^2 + 1) + \theta x}{\theta^2 + 1} e^{-\theta x} \right] \left[ 1 - \left( \frac{(\theta^2 + 1) + \theta x}{\theta^2 + 1} e^{-\theta x} \right)^2 \right]^{\alpha-1}; x > 0, \theta > 0 \quad (16)$$

Where:

- $g_{TL-sh}(x; \alpha, \theta)$ : the probability density function (PDF) of the proposed model (TL-Sh).
- $\alpha$ : Shape parameter.
- $\theta$ : Scale parameter.

### 2.4.2 Topp-Leone–Rayleigh Distribution

The random variable X follows the Topp-Leone Rayleigh distribution, and its probability density function (PDF) is represented by the following equation [19]:

$$g_{TL-RR}(x, \alpha, \beta, \theta) = \theta \frac{x^3}{\beta^4 \alpha^2} e^{-\frac{x^4}{4\beta^4 \alpha^2}} \left(1 - e^{-\frac{x^3}{8\beta^4 \alpha^2}}\right)^{\theta-1} \left(1 + e^{-\frac{x^3}{8\beta^4 \alpha^2}}\right)^{\theta-1}, x > 0, \alpha, \beta, \theta > 0 \dots (17)$$

Where:

- $g_{TL-RR}(x; \alpha, \theta, \beta)$ : the probability density function (PDF) of the model (TL-RR).
- $\theta, \alpha$ : Shape parameters.
- $\beta$ : Scale parameter.

### 2.5 Practical Aspect

The probability distributions (Shanker, Rayleigh, Topp-Leone Shanker, and Topp-Leone Rayleigh Rayleigh) were estimated using the Maximum Product of Spacings (MPS) method. The Table 1 summarizes the research sample data, representing the demand quantities (11/250 KV transformers), consisting of 36 observations for the period 2019–2021.

Table 1: Demand quantities (11/250 KV transformers) for Diyala general company for the years 2019–2021.

171.2	288	243.2	465.6	476.8	168
187.2	384	444.8	265.6	249.6	472
372.8	467.2	248	172.8	321.6	164.8
323.2	219.2	185.6	326.4	187.2	169.6
164.8	163.2	296	310.4	216	174.4
180.8	339.2	398.4	176	451.2	475.2

Table 2: Descriptive statistics of demand quantities (11/250 KV transformers) for Diyala general company for the years 2019–2021.

Statistics	Value
Mean	286.667
Variance	12208.1
Skewness	0.486425
Kurtosis	1.80855
Median	249.6
Standard Deviation	110.49

The results in Table 2 indicate that the distribution is positively skewed, as the sample mean (286.667) is greater than the median (249.6). This is further illustrated by the distribution shape shown in Figure 3 and the positive skewness value (0.486425).

It can be observed from Table 3 that the Topp-Leone–Shanker (TL-Sh) distribution is superior to the baseline Sh distribution and the Topp-Leone–Rayleigh (TL-RR) distribution, as it possesses the lowest values for the goodness-of-fit criteria (AIC, AICc, BIC). Accordingly, it is more flexible in representing the actual data of the study sample summarized in Table 1.

As observed from the results of the goodness-of-fit tests shown in Table 5, the study sample data adequately fit the proposed TL-Sh distribution. This is due to the fact that the p-values for all three tests are greater than the significance level set for the tests ( $\alpha=0.05$ ). The comparative goodness-of-fit criteria for the considered distributions are presented in Table 4.

Table 3: Parameter estimation the distributions are (TL-RR), (TL-Sh), and (Sh).

Model	Parameter Estimation		
	Theta	Beta	Alpha
TL-RR	0.652337	45.829	32.7123
TL-Sh	0.007789		4.72153
Sh	0.006846		0.006846

Table 4: Results of comparative standards between (TL-RR), (TL-Sh), and (Sh).

Dist	AIC	AICc	BIC
TLG-RR	445.825	446.075	450.575
TLSH	439.79	439.972	442.957
S	458.881	458.94	460.465

Table 5: Goodness-of-fit tests for the TL-Sh distribution of demand quantities.

TEST	Statistic	P-Value
Anderson-Darling	1.35247	0.215121
Cramér-von Mises	0.183627	0.301568
Kolmogorov-Smirnov	0.18461	0.151295

Figure 3 further illustrates the suitability of the proposed Topp-Leone distribution (TL-Sh) compared to the original Shanker distribution (Sh) and the TL-RR distribution.

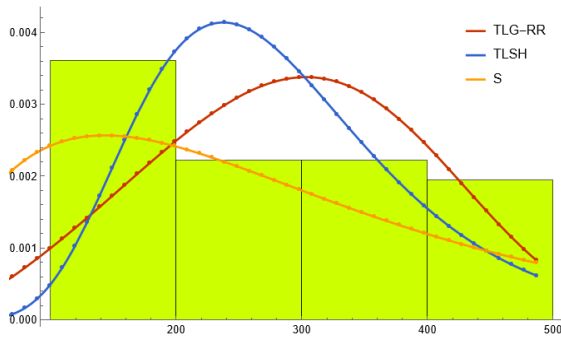


Figure 3: Histogram of the real data.

### 2.6 Probabilistic Inventory Model Indicators (EOQ)

After verifying the suitability of the study sample data and estimating the parameters of the Topp-Leone Shanker and Topp-Leone Rayleigh distributions from the sample data, the probabilistic inventory model indicators for each distribution were calculated and summarized in Table 5.

Table 6: Probabilistic inventory model indicators (EOQ).

Indicators	(TL-Sh)	(TL-RR)
$\hat{Y}$	1951.848	627.401
$\tilde{Y}$	6785.396	6785.396
Reorder Point	247.304	526.448
Optimal Order Quantity	75.037	66.578
Expected Shortage	75.195	297.137
Expected Inventory Holding Cost	71811.012	4714928.335
Expected Shortage Cost	118878187.3	529437455.2
Expected Ordering Cost	2121.682	24193.826
Total Expected Cost	118952120	534176577.4

From the analysis of the inventory indicators presented in Table 6, the following observations can be made:

- When the inventory level 247.304 transformers, the optimal replenishment quantity is 75.037 units. The expected ordering cost amounts to 2,121.682 IQD, and the expected shortage equals 75.195 transformers, with an expected shortage cost of 118,878,187.3 IQD. Additionally, the expected holding cost is 71,811.012 IQD, leading to an expected total cost of 118,952,120 IQD.
- On the other hand, the inventory model indicators for the probabilistic TL-RR distribution indicate that when the inventory

level reaches 526.448 electrical transformers, the optimal replenishment quantity is 66.578 units. The expected ordering cost is 24,193.826 IQD, and the expected shortage amounts to 297.137 transformers, with an expected shortage cost of 529,437,455.2 IQD. Moreover, the expected holding cost is 4,714,928.335 IQD, resulting in an expected total cost of 534,176,577.4 IQD.

- The total expected cost under the Topp-Leone-Shanker (TL-Sh) distribution is lower than that under the Topp-Leone-Rileyleigh-Rileyleigh (TL-RR) distribution. Therefore, it can be concluded that the probabilistic inventory model based on the TL-Sh distribution performs better than the TL-RR distribution in estimating the inventory model indicators.

## 3 RECOMMENDATIONS

Based on the above conclusions, the following recommendations are proposed:

- 1) To utilize the proposed probabilistic distribution (TL-Sh) in inventory modeling, alongside Bayesian estimation methods, in order to achieve more accurate estimations.
- 2) Relying on the probabilistic inventory model allows for improved inventory control, as it is characterized by accuracy and adaptability to data variations.
- 3) Employing modern scientific methods to determine optimal production quantities and establish annual production plans, rather than relying solely on personal experience and skills.

## 4 CONCLUSIONS

Based on the results of the practical analysis above, the following conclusions can be drawn:

The optimal order quantity for the Topp-Leone Shanker distribution is 75.037, and for the Topp-Leone Rayleigh Rayleigh distribution is 66.578. These quantities are determined to ensure inventory availability to meet demand while simultaneously minimizing storage and spoilage costs. The total cost associated with the Topp-Leone Shanker distribution is 118952120 Iraqi Dinars, whereas the total cost for the Topp-Leone Rayleigh Rayleigh distribution is 534176577.4 Iraqi Dinars. The sample data (monthly demand quantities) are fitted to the Topp-Leone Shanker probability distribution because the p-value

(significance level) of the goodness-of-fit tests is greater than the pre-specified significance level ( $\alpha=0.05$ ). The proposed probabilistic distribution, Topp-Leone Shanker (TL-Sh), is the most suitable distribution compared to the Shanker (Sh) distribution for representing the demand quantities for the production of electrical transformers, according to the statistical criteria (AIC, AICc, BIC). The use of probabilistic models in inventory management is effective in determining the optimal production quantities and reorder points, which helps reduce the overall inventory costs for the company.

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