Neutrosophic Multi-Criteria Assignment Models for Industrial Decision-Making

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Abstract: One of the most useful decision-making problems is known as Assignment Problem which used in different real-life situations. Solving single and Multi-Criteria Assignment Problems in fuzzy and neutrosophic environments have received considerable attention recently. In this Paper we use integrating Ranking Function (RF) and Weighted Goal Programming (WGP) to solve multi-criteria assignment problems in neutrosophic environments. Goal Programming models formulate these criteria as goals to be optimized within resource constraints. This approach allows for a more comprehensive and efficient allocation of resources by considering both the priorities of different objectives and the limitations of available resources. The presented approach has used to solve a multi-criteria and unbalanced trapezoidal neutrosophic assignment problem that consists of the four main steps: the first step is to transform the trapezoidal neutrosophic numbers into crisp numbers using ranking function formula, the second step is to identify the criteria and its weights, the third step is to identify assign matrix that consist of rows present the activities (jobs) and columns represent allocation of resources (machines) and formulate mathematical model for weighted goal programming. Final step solves proposal model and find the optimal resource allocation that minimizes deviations from the defined goals. This work has presented example of multi-criteria and unbalanced trapezoidal neutrosophic assignment problem which represents data for the Heavy Engineering Equipment State, which allows us to verify its effectiveness in a real industrial environment that has three criteria (rolling time, rolling cost, and number of rolling passes), three machines (allocation of resources) and eight job (activities). The results show that the presented approach is an efficient and easy to implement and can extend to other related problems.

1 INTRODUCTION

Multi-Criteria Decision Making (MCDM) is an extensive field dedicated to selecting the best alternative based on multiple criteria. The application of LP and WGP in MCAP enables decision-makers to account for the relative importance of different criteria, thus leading to more informed and balanced decisions in complex scenarios. More details about Linear programming LP and WGP we refer to [1]-[3].

Yazdani et al. (2019) [4] discuss the necessity of determining criteria weights in MCDM techniques, asserting that the effectiveness of WGP relies heavily on how well these weights are assigned [5]. This emphasis on criteria weights indicates that WGP is adaptable to various contexts, enhancing its relevance in solving MCAPs. Moreover, the study addresses the influence of both subjective and objective weighting methods, suggesting that WGP can accommodate different decision-making preferences [6].

The flexibility of WGP to incorporate subjective and objective weighting methods is particularly beneficial in real-world applications where decision-makers often prefer straightforward approaches. This ease of computation for subjective methods signifies practical implications for WGP, allowing for a more efficient assignment process [7].

The balanced allocation problem, a traditional allocation problem, has received considerable research attention. Several studies suggest that alternative methods exist for solving this problem besides the single stage methods. Meanwhile, Haleemah et al. (2021) [8] introduced a penalty method to handle the assignment problem with a maximization objective function. The outcomes of the solution matched the results of the Hungarian method's optimal solution. Sadiq et al. (2022) [9] investigated the assignment problem using three techniques: the Hungarian method, the alternative technique, and the new technique proposed by Haleemah et al. Taufik et al. (2023) [10] utilized integer linear programming and the Hungarian technique using Production and Operations Management Quantitative Methods (POM QM) software to solve assignment problems. Vásconez et al. (2024) [11] proposed a smart delivery system to optimize the task assignment problem for delivery personnel with pending orders. The unbalanced assignment problem occurs when the number of activities is not equal to the number of resources, requiring modifications to the single stage method, such as assigning multiple activities to resources or using alternative methods.

Jain et al. (2020) [12] developed an unbalanced time minimization assignment problem using a priority system for projects completed in two stages, with some primary jobs completed in the first stage and the remaining jobs starting after. Anandhi and Geetha (2020) [13] introduced the "Maximum Average with Minimum Cost Method (MAMCM)" for solving assignment problems, which offers the best solution with minimal computation, tested on balanced and unbalanced assignments. However, Wulan et al. (2020) [14] utilized the Kotwal Dhope Method to reduce unbalanced assignments in online higher education courses, with assistance from Hungarian and Matrix One. The study found the proposed method the most cost effective option. Although the authors Rabbani et al. (2020) [15] used a modified Hungarian method with included time parameters to manage job distribution to machines, a numerical example was used. In (2021), Rezaul [16] employed an approach written in Python combined with the Hungarian method to solve balanced and unbalanced assignment problems, achieving similar outcomes to the Hungarian method.

The research above related to the single criteria assignment problem, while the most problem today use multi- criteria in solving assignment problem.

Sheela (2022) [17] introduced a deviation assignment method for solving multi objective problems, comparing it with the Hungarian method and confirming its optimality through numerical examples.

Liu and Yong (2022) [18] used the multi objective decision making mathematical model to optimize performance appraisal in unbalancing human resource allocation. It consists of two objectives: the maximum quality and the minimum time to complete the task. The superior leader determined the objectives' weights, and the problem was solved using the Hungarian method. Tran and Nguyen (2023) [19] investigated and solved two cases of the assignment problem, balanced and unbalanced, solved by the Hungarian method. In the first case, the wage costs were distributed among five employees for five tasks. In the second case, four employees carried out five tasks with three objectives, high quality, and short completion time with the lowest risk (weighting by managers).

While extensive research exists on single-criterion assignment problems including balanced, unbalanced, and time-minimization variants the growing complexity of real-world decision-making necessitates multi-criteria approaches. Recent studies have begun addressing Multi-Criteria Assignment Problems (MCAP), yet many rely on multi-stage processes or require simplification into a single-criterion model, especially under uncertain environments such as neutrosophic conditions. For instance, methods combining Hungarian with weighting or deviation-based approaches still involve sequential optimization steps, increasing computational time and potential information loss. This reveals a clear scientific gap: the absence of an efficient single-stage method capable of directly solving neutrosophic MCAP without criterion conversion. To address this, we propose the RF-WGP approach, which integrates ranking functions and weighted goal programming in a unified framework, enabling direct, time-efficient, and accurate solutions for neutrosophic multi-criteria assignments. The most important drawback for the solution in the proposed methodology in reference [20] in multiple stages and its need to converted a multi-criteria assignment problem into a single- criteria assignment problem.

2 PRELIMINARIES

In this section, some basic concepts for neutrosophic set and numbers are presented. These concepts are taken from [21]-[26].

2.1 Basic Definitions

Definition 1 [21]: Let E be a universe. A single valued neutrosophic set A over E, which can be used in real scientific and engineering applications, is an neutrosophic set, where the truth-membership function, indeterminacy-membership function and falsity- membership function are respectively is defined by: $T_A: E \rightarrow [0,1]$, $I_A: E \rightarrow [0,1], and\ F_A: E \rightarrow [0,1]$, such that $0 \leq T_A + I_A + F_A \leq 3$.

Defintion 2 [21]: The trapezoidal neutrosophic nuber A is a neutrosophic set in R with the following truth, indeterminacy and falsity membership functions:

$$T_A(x) = \begin{cases} \frac{a_A(x - a_1)}{a_2 - a_1} : a_1 \leq x \leq a_2 \\ a_A \quad : a_2 \leq x \leq a_3 \\ a_A\left(\frac{x - a_3}{a_A - a_3}\right) : a_3 \leq x \leq a_4 \\ 0 \quad otherwise. \end{cases}$$

$$I_A(x) = \begin{cases} \frac{\left(a_2 - x + \theta_A(x - \acute{a}_1)\right)}{a_2 - a_1} : \acute{a}_1 \leq x \leq a_2 \\ \theta_A \quad : a_2 \leq x \leq a_3 \\ \frac{\left(x - a_3 + \theta_A(\acute{a}_4 - x)\right)}{a_4 - a_3} : a_3 \leq x \leq \acute{a}_4 \\ 1 \quad otherwise. \end{cases}$$

$$F_A(x) = \begin{cases} \frac{\left(a_2 - x + \beta_A(x - \acute{a}_1)\right)}{a_2 - a_1} : \acute{a}_1 \leq x \leq a_2 \\ \beta_A \quad : a_2 \leq x \leq a_3 \\ \frac{\left(x - a_3 + \beta_A(\acute{a}_4 - x)\right)}{a_4 - a_3} : a_3 \leq x \leq \acute{a}_4 \\ 1 \quad otherwise. \end{cases}$$

2.2 Ranking Functions

Ranking function is important tool that used to transform fuzzy number or neutrosophic number into crisp number so that an efficient method can be applied easily [27], [28] present new and efficient ranking function as follows.

$$R(\tilde{a}) = \left|\left[\frac{(a^l + a^u) - 3(a^{m_1} + a^{m_2})}{-4}\right] * (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})\right| \quad (1)$$

Where, $\tilde{a} = \{a^l, a^{m_1}, a^{m_2}, a^u\}$ is a trapezoidal neutrosophic number and $(T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ are the truth, indeterminacy and falsity degrees of this number.

The choice of the presented ranking function is justified as follows: In neutrosophic decision-making problems, objective functions and constraints are commonly expressed in terms of neutrosophic numbers, which involve truth, indeterminacy, and falsity components. However, classical optimization and assignment models require crisp numerical values. Therefore, a ranking function is an essential tool for converting neutrosophic numbers into comparable crisp values.

In this paper, the ranking function proposed in [28] is adopted because it simultaneously considers the truth (T), indeterminacy (I), and falsity (F) degrees of a trapezoidal neutrosophic number. Unlike conventional ranking approaches that rely solely on truth membership, this function explicitly penalizes indeterminacy and falsity, leading to a more balanced and realistic evaluation of uncertain information.

Moreover, the selected ranking function preserves the geometric characteristics of trapezoidal neutrosophic numbers and produces a single representative crisp value without losing essential uncertainty information. This makes it particularly suitable for multi-criteria assignment problems, where consistent comparison among alternatives is required. Its effectiveness and computational simplicity have also been demonstrated in previous neutrosophic optimization studies, which further justifies its selection in the proposed RF-WGP framework.

3 NEUTROSOPHIC ASSIGNMENT PROBLEM

The assignment problem is a special category of transportation problem that deals with assigning tasks to machines and personnel to locations. Assignment problems are often described as applications that are used to assign "activities" to "resources" (or jobs to machines, tasks to employees, etc.) within certain constraints to keep the total cost as low as possible [29].

Also, the assignment problem is one of the fundamental challenges in operations research, since it finds the arrangement that works best [30]. In one –to-many assignment problems

The decision maker is assigning more than one activity to one resource; the resource can achieve more than one activity [31]. The One-to-Many assignment (Called Group Role Assignment) [32]. Some cases of One-to-Many assignment problems such as unbalanced assignment problem which raised when the quantity of resources doesn't match the quantity of tasks [33]. Generalized assignment problem: This problem enables the possibility of assigning multiple activities to a resource while accounting for the amount of the resource's capacity that each activity would require. Each activity in the problem will be assigned to a single resource [34].

More details for assignment problem can found in [35]-[37].

The field of industrial engineering has expanded to include multi-criteria decision-making (MCDM), which focuses on creating mathematical and computational tools to assist decision-makers in their subjective assessment of performance criteria [38], [39].

Three primary techniques are used in MCDM (pairwise comparison, outranking, and distance-based) [40]. The Analytical Hierarchy Process (AHP) and Analytic Network Process (ANP) are techniques for examples pairwise comparison [41], [42].

Examples of outranking techniques include the Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), which relies on comparison to determine the extent to which one alternative is preferred over another and produces a comprehensive ranking of the alternatives [43], [44].

Through the use of distance-based techniques, the best solution is determined by how close a solution is to the ideal point. Several popular distance-based methods as in [45], [46].

An approach has emerged as a significant method for addressing MCAP by allowing decision-makers to prioritize various criteria through the assignment of weights known as The Weighted Goal Programming (WGP). Recently, netrosophic assignment problem has studied and the objective of this problem is to minimize the total netrosophic cost of assigning all the jobs to the available machines (one machine per job) at the least total cost. For more details we refer to [47]-[51].

4 WEIGHTED GOAL PROGRAMMING (WGP)

Goal programming is a solution method commonly used in practice to convert a multiple objective program into a solvable single objective program. Goal programming requires that the decision maker specifies a goal for each criterion. [52].

The mathematical for preemptive goal programming is given as [52]:

$$Min\tilde{a} = \begin{Bmatrix} P_1[g_1(d_1^+, d_1^-)], P_2[g_2(d_2^+, d_2^-)], ., \\ .P_k[g_k(d_k^+, d_k^-)] \end{Bmatrix} \quad (2)$$

S.t.

$$\sum_{i=1}^{m} C_{in}X_n + d_i^- - d_i^+ = b_i \quad (3)$$

$$\sum_{i=1}^{m} C_{in}X_n \geq, =, \leq b_i \quad (4)$$

$$X_j, d_i^+, d_i^- \geq 0, \; n = 1,2,3, \dots, N. \quad (5)$$

Where, (4) represent Structural Constraints in goal programming model.

Weighted Goal Programming model consist of multiple objective simultaneously and to assign goals for each objective so that the solution to each goal is achieved. The mathematical model of a WGP is given as [53]:

$$\text{Min}\tilde{a} = \begin{Bmatrix} w_1[g_1(d_1^+, d_1^-)] + w_2[g_2(d_2^+, d_2^-)].. \\ +w_k[g_k(d_k^+, d_k^-)] \end{Bmatrix} \quad (6)$$

S.t.

$$\sum_{i=1}^{m} C_{in}X_n + d_i^- - d_i^+ = b_i \quad (7)$$

$$\sum_{i=1}^{m} C_{in}X_n \geq, =, \leq b_i \quad (8)$$

$$X_j, d_i^+, d_i^- \geq 0, \; n = 1,2,3, \dots, N. \quad (9)$$

Where w_i represent weighted for goal.

In Weighted Goal Programming, the target value g_i epresents the aspiration level associated with the i-th objective and plays a crucial role in guiding the optimization process. In the proposed approach, the target values are determined after transforming the neutrosophic objective functions into crisp forms using the ranking function. Specifically, for each criterion, the corresponding crisp objective function is first evaluated independently under the given system constraints. The optimal or most desirable value obtained from this evaluation is then used as the target value g_i.

This procedure ensures that the targets are realistic, attainable, and consistent with the decision environment. In cases where decision makers have prior preferences or practical benchmarks, these values can also be incorporated as target levels. The deviations from these targets are minimized using weighted penalties, where the assigned weights reflect the relative importance of each goal. This allows the WGP model to handle multiple conflicting objectives simultaneously while maintaining flexibility in prioritization. Thus, the integration of ranking functions with WGP provides a coherent methodological framework: the ranking function handles neutrosophic uncertainty, while WGP manages multiple objectives through well-defined target values and priority weights.

Table 1: Example of Neutrosohpic Data of heavy engineering equipment state company.

Rolling Time			
Job	Machine 1	Machine 2	Machine 3
1	(54,55,56, 57)	(45,46,47,48)	(42,43,44,45)
2	(29,30,31, 32)	(24,25,26,27)	(21,22,23,24)
3	(19,20,21, 22)	(19,20,21,22)	(18,19,20,21)
4	(29,30,31,32)	(27,28,29,30)	(24,25,26,27)
5	(13,14,15,16)	(13,14,15,16)	(12,13,14,15)
6	(37,38,39,40)	(33,34,35,36)	(29,30,31,32)
7	(38,39,40,41)	28,29,30,31)	(24,25,26,27)
8	(17,18,19,20)	15,16,17,18)	(14,15,16,17)
Job	Rolling Cost		
1	(95,96,97,98)	(80,81,82,83)	(44,45,46,47)
2	(28,29,30,31)	(21,22,23,24)	(20,21,22,23)
3	(15,16,17,18)	(16,17,18,19)	(19,20,21,22)
4	(27,28,29,30)	(12,13,14,15)	(21,22,23,24)
5	(11,12,13,14)	(13,14,15,16)	(19,20,21,22)
6	(44,45,46,47)	(35,36,37,38)	(26,27,28,29)
7	(58,59,60,61)	(37,38,39,40)	(27,28,29,30)
8	(14,15,16,17)	(15,16,17,18)	(19,20,21,22)
Job	Rolling Iteration		
1	(11,12,13,14)	(2,3,4,5)	(4,5, 6,7)
2	(4,5,6,7)	(3,4,5,6)	(3,4,5,6)
3	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)
4	(4,5,6,7)	(4,5, 6,7)	(3,4,5,6)
5	(3,4,5,6)	(3,4,5,6)	(3,4,5,6)
6	(5,6,7,8)	(4,5, 6,7)	(3,4,5,6)
7	(7,8,9,10)	(5,6, 7,8)	(4,5, 6,7)
8	(3,4,5,6)	(3,4, 5,6)	(3,4,5,6)

5 EXAMPLE

5.1 Neutrosohpic Data

To show the presented computational approach, we introduce the heavy engineering equipment State company in neutrosophic environment based on the crisp data as in [20] Where, the neutrosophic data consist of three criteria (rolling time, rolling cost, and number of rolling passes), three machines (allocation of resources) and eight job (activities) as shown in Table 1.

The weights of criteria are w_1=0.47, w_2=0.35, w_3=0.18 for rolling time, rolling cost, and number of rolling passes), respectively.

5.2 Implementation of Integrated (RF-WGP) Approach

This approach using to solving a multi-criteria and unbalanced neuromorphic assignment problem that consists of the four main steps as following:

The first step: transform all trapezoidal neutrosohic numbers in Table 1 into crisp numbers using ranking function in (1) with (0.8,0.1,0.1) confirmation degree as in Table 2, Table 3, and Table 4.

The second step: identify the criteria and its weights, where determine the weights of criteria are w_1=0.47, w_2=0.35, w_3=0.18 for rolling time, rolling cost, and number of rolling passes), respectively.

The third step: identify assign matrix that consist of rows present the activities (jobs) and columns represent allocation of resources (machines) as shown in Table 1, then formulate proposal weighted goal programming to find the optimal resource allocation when unbalanced assignment problem m > n, where the number jobs is smaller than the number of machines as following:

$$Min\ Z = \begin{Bmatrix} w_1[g_1(d_1^+, d_1^-)] + w_2[g_2(d_2^+, d_2^-)] \\ +..+w_3[g_k(d_3^+, d_3^-)] \end{Bmatrix} \quad (10)$$

S.t.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} + d_i^- - d_i^+ = g_i \ for\ all\ criteria \quad (11)$$

$$\sum_{j=1}^{3} X_{ij} = 1 \ for\ all\ i = 1,2,\dots,m \quad (12)$$

$$\sum_{j=1}^{8} X_{ij} \ge \alpha \ for\ all\ j = 1,2,\dots,n \quad (13)$$

$$\sum_{i=1}^{8}\sum_{j=1}^{3} X_{ij} = \beta \quad (14)$$

$$\alpha = \left\lfloor \frac{m}{n} \right\rfloor \quad (15)$$

$$\beta = Max(m,n) \quad (16)$$

$$X_{ij}, d_i^-, d_i^+, \alpha, \beta \ge 0 \quad (17)$$

$$X_{ij} = binary\ variables(0\ or\ 1). \quad (18)$$

Where:
- g_i: represent target for criteria *i*;
- w_i: represent weighted for all criteria;
- X_{ij}:confactor for decision variables;
- C_{ij}:decision variables;
- d_i^-,d_i^+:deviational variables.

Equation (10) shows the assignment problem's objective function, assigning the activities to the resources. Equation (11) represents the criteria constraints that add to it deviation variables. Equation (12) represents the activity(job) constraints where each must be accomplished, therefore equal to one, while (13) shows the resource constraints where each resource must be assigned more than or equal to α, where $\alpha = \left\lfloor \frac{m}{n} \right\rfloor = \left\lfloor \frac{8}{3} \right\rfloor = 2$ as shown in (15). Equation (14) shows that the sum of the activities assigned to the resources that should equal to maximum (total number of activities or total number of resource), where $\beta = Max(\ m,n) = Max(\ 8,3) = 8$ shown in (15). The (17) shows the non-negative constraint for variables, (18) shows the binary for decision variables.

The fourth step: solve proposal weighted goal programming and find the optimal resource allocation when number of job=8, number of machine=3, number of criteria=3, the right side for goal constraints are 117.6, 124.2, 22.8 respectively represent least sum of time, cost and iterations values for each machine, therefore the mathematical model as following:

$$Min\ Z = \ 0.47d_1^+ + 0.35d_2^+ + 0.18d_3^+$$

S.t.

$33.3X_{11} + 27.9X_{12} + 26.1X_{13} + 18.3X_{21} + 15.3X_{22} + 13.5X_{23} + 12.3X_{31} + 12.3X_{32} + 11.7X_{33} + 18.3X_{41} + 17.1X_{42} + 15.3X_{43} + 8.7X_{51} + 8.7X_{52} + 8.1X_{53} + 23.1X_{61} + 20.7X_{62} + 18.3X_{63} + 23.7X_{71} + 17.7X_{72} + 15.3X_{73} + 11.1X_{81} + 9.9X_{82} + 9.3X_{83} + d_1^- - d_1^+ = 117.6$

$57.9X_{11} + 48.9X_{12} + 27.3X_{13} + 17.7X_{21} + 13.5X_{22} + 12.9X_{23} + 9.9X_{31} + 10.5X_{32} + 12.3X_{33} + 17.1X_{41} + 8.1X_{42} + 13.5X_{43} + 7.5X_{51} + 8.7X_{52} + 12.3X_{53} + 27.3X_{61} + 21.9X_{62} + 16.5X_{63} + 35.7X_{71} + 23.1X_{72} + 17.1X_{73} + 9.3X_{81} + 9.9X_{82} + 12.3X_{83} + d_2^- - d_2^+ = 124.2$

$7.5X_{11} + 2.1X_{12} + 3.3X_{13} + 3.3X_{21} + 2.7X_{22} + 2.7X_{23} + 2.7X_{31} + 2.7X_{32} + 2.7X_{33} + 3.3X_{41} + 3.3X_{42} + 2.7X_{43} + 2.7X_{51} + 2.7X_{52} + 2.7X_{53} + 3.9X_{61} + 3.3X_{62} + 2.7X_{63} + 5.1X_{71} + 3.9X_{72} + 3.3X_{73} + 2.7X_{81} + 2.7X_{82} + 2.7X_{83} + d_3^- - d_3^+ = 22.8$

$$X_{11} + X_{12} + X_{13} = 1$$
$$X_{21} + X_{22} + X_{23} = 1$$
$$X_{31} + X_{32} + X_{33} = 1$$
$$X_{41} + X_{42} + X_{43} = 1$$
$$X_{51} + X_{52} + X_{53} = 1$$
$$X_{61} + X_{62} + X_{63} = 1$$
$$X_{71} + X_{72} + X_{73} = 1$$
$$X_{81} + X_{82} + X_{83} = 1$$

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} + X_{71} + X_{81} \ge 2$
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} + X_{72} + X_{82} \ge 2$
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} + X_{73} + X_{83} \ge 2$

$X_{11} + X_{12} + X_{13} + X_{21} + X_{22} + X_{23} + X_{31} + X_{32} + X_{33} + X_{41} + X_{42} + X_{43} + X_{51} + X_{52} + X_{53} + X_{61} + X_{62} + X_{63} + X_{71} + X_{72} + X_{73} + X_{81} + X_{82} + X_{83} = 8$
$X_{ij} = binary\ variables(0\ or\ 1)$

Solving the above model where the result as following:

$X_{13} = X_{22} = X_{31} = X_{43} = X_{51} = X_{63} = X_{73} = X_{82} = 1$
$d_1^- = 0,\ d_1^+ = 3.6, d_2^- = 9, d_2^+ = 0, d_3^- = 0, d_3^+ = 0,$
$Z = 0.47(\ d_1^+ = 3.6) + 0.35(d_2^+ = 0) + 0.20\ (d_3^+ = 0) = 1.692$

Table 2: Crisp values of rolling time.

Job	Machine 1	Machine 2	Machine 3
1	33.3	27.9	26.1
2	18.3	15.3	13.5
3	12.3	12.3	11.7
4	18.3	17.1	15.3
5	8.7	8.7	8.1
6	23.1	20.7	18.3
7	23.7	17.7	15.3
8	11.1	9.9	9.3

Table 3: Crisp values of rolling cost.

Job	Machine 1	Machine 2	Machine 3
1	57.9	48.9	27.3
2	17.7	13.5	12.9
3	9.9	10.5	12.3
4	17.1	8.1	13.5
5	7.5	8.7	12.3
6	27.3	21.9	16.5
7	35.7	23.1	17.1
8	9.3	9.9	12.3

Table 4: Crisp values of rolling iteration.

Job	Machine 1	Machine 2	Machine 3
1	7.5	2.1	3.3
2	3.3	2.7	2.7
3	2.7	2.7	2.7
4	3.3	3.3	2.7
5	2.7	2.7	2.7
6	3.9	3.3	2.7
7	5.1	3.9	3.3
8	2.7	2.7	2.7

5.3 Results and Discussion

To ensure transparency and reproducibility, the comparison with the method in [20] is conducted using the same original problem data, identical job–machine configurations, and the same evaluation criteria (assignment time, assignment cost, and assignment iteration). This guarantees that any observed differences in results are due solely to methodological differences rather than data inconsistencies.

First, the proposed RF-WGP model is applied using trapezoidal neutrosophic numbers, and the resulting assignments and aggregated performance measures are reported in Table 5. Subsequently, for a fair comparison with [20], the same assignments are evaluated using the original crisp data, following the solution structure reported in Table 2 of reference [20]. These results are summarized in Table 6. Finally, Table 7 presents a direct side-by-side comparison between the proposed model and the method from [8].

This step-by-step comparison framework ensures that the results are reproducible and that the evaluation criteria are consistently applied across both methods.

The result shown the jobs (3 and 5) assign on machine1, jobs (2 and 5) assign on machine2 and, jobs (1,4,6 and 7) assign on machine3, can be summary assignment results as following Table 5 depend on trapezoidal neutrosophic number.

The comparison between the result for proposal model and the result reference [20], needs original data depend on the solution in Table 2 as following in Table 6.

Table 5: Summary the result for proposal model depends on trapezoidal neutrosophic number.

Machines	Jobs	Assignment Time	Assignment Cost	Assignment Iteration
Machine 1	Job3, Job5	12.3+8.7=21	9.9+7.5=17.4	2.7+2.7=5.4
Machine 2	Job2, Job8	15.3+9.9=25.2	13.5+9.9=23.4	2.7+2.7=5.4
Machine 3	Job1, Job4 Job6, Job7	26.1+15.3+18.3+15.3=75	27.3+13.5+16.5+17.1=74.4	3.3+2.7+2.7+3.3=12
Total for all criteria		117.6	124.2	22.8
Target criteria		121.2	115.2	22.8
deviational variables		+3.6	-9	0

Table 6: Summary of the result for proposal model depend on original data.

Machines	Jobs	Assignment Time	Assignment Cost	Assignment Iteration
Machine 1	Job3, Job5	20+14=34	16+12=28	4+4=8
Machine 2	Job2, Job8	25+16=41	22+16=38	4+4=8
Machine 3	Job1, Job4 Job6, Job7	43+25+30 +25=123	45+22+27 +28=122	5+4+4+5= 18
Total for all criteria		117.6	198	188
Target criteria		121.2	192	205
deviational variables		+3.6	+6	-7

The key methodological difference between the proposed approach and the method in [20] lies in the solution structure. The method in [20] follows a multi-stage process, where neutrosophic data handling, defuzzification, and optimization are treated as separate sequential steps. This separation can increase computational effort and may propagate approximation errors from one stage to the next. In contrast, the proposed RF-WGP approach operates as a single-stage integrated framework, where: Neutrosophic uncertainty is handled directly through an appropriate ranking function, and Multiple conflicting objectives are simultaneously optimized using Weighted Goal Programming.This integration allows all criteria to be considered concurrently within a unified optimization model, reducing intermediate transformations and improving computational efficiency.

Table 7: Comparison between the result for the proposal model and the result from reference [20].

The model	Total Assignm ent Time	Total Assignment Cost	Total Assignment Iteration
Proposal model	198	188	34
Reference [20]	207	188	36
The different between two model	9	0	2

The results shown that our proposal model is best from the result reference [20], as following Table 7. As shown in Table 7, the proposed model achieves:

A lower total assignment time compared to reference [8], The same total assignment cost, and A reduction in total assignment iterations.

These results indicate that the proposed single-stage RF-WGP method provides at least equal, and in some cases superior, performance compared to the multi-stage approach in [20]. The reduction in assignment time and iterations highlights the practical advantage of solving the problem in a single integrated stage, especially for large-scale or time-sensitive applications.

6 CONCLUSIONS

The integration of the ranking function and the weighted goal programming approach within the multi-criteria neutrosophic assignment problem framework provides a robust methodology for enhancing decision-making processes under uncertainty and indeterminacy. Also, helps the decision maker to handle the imprecise and inconstant data which are common in real world applications.

The proposed approach was tested using an example of three-criteria neutrosophic assignment problem which represents a neutrosophic extension of the data set presented in [20].

The results showed that the proposed method is more effective in reducing total assignment time and total assignment cost compared to existing approaches.

It also the neutrosophic assignment problems are solved in single stage and do not require converting the multi-criteria assignment problem into a single-criteria assignment problem. This work opens new direction for future studies which can help the decision maker to reach the optimal decision. Further work may focus on extend this research using new ranking functions, and other related real-life applications such as project scheduling, resources allocation, and supply chain management.

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