Robust and Nonparametric Estimation of the Pareto Reliability Function

Sarah Majeed Fenjan[1], Sarah Adel Madhloom[2], Ali Abed Hasan[3] and Hayder Raaid Talib[1]
[1]*University of Sumer, College of Administration & Economics, 57009 Umm Qasr, Iraq*
[2]*Middle Technical University Suwaira Technical Institute, 52002 Suwaira, Iraq*
[3]*Imam AL- Kadhum College, 52001 Baghdad, Iraq*
sarah.majeed@uos.edu.iq, sarah_adel@mtu.edu.iq, ali.abdhassan@iku.edu.iq, hayder.raaid@uos.edu.iq

Keywords: Pareto Distribution, Reliability Function, MM-Estimate, Kernel Estimation Method.

Abstract: The research addresses the topic of reliability, which is defined as the probability of any component of a system being completed within a specified period of time and under the same conditions. The reliability function and failure function of the Pareto distribution were studied. Several methods exist for estimating the reliability function, but the conditions for most of these methods are often missing in the data, such as abnormal, extreme, or unacceptable values. Two estimation methods (a robust method and a nonparametric method) were used: the MM-estimate and the nonparametric kernel estimation. After these two estimation formulas were derived to reach their capacity, a comparison was made between these estimates using simulation experiments with different sample sizes (10, 30 and 60). Each experiment was repeated 1,000 times to achieve the objective, and the results were compared using the mean square error (MSE) criterion. Through the results obtained, it was found that the best estimation method is the (MM-estimate) method, where it was estimated that the reliability function gradually decreases with time, which is consistent with the characteristics of this function.

1 PARETO DISTRIBUTION

This distribution is attributed to the Swiss-born Italian economist Vilfredo Pareto (1848–1923), who laid the foundations of the Pareto distribution in economics by studying the distribution of incomes exceeding a known threshold k. The general Pareto distribution, sometimes called the first or traditional Pareto distribution, has important applications in economics, communications, and various engineering sciences [1], [2].

It is assumed that (x_1, x_2) follow a Pareto distribution with parameters (α_i, k_i), where i = 1,2, α_i is the shape parameter, and k_i is the scale parameter, sometimes interpreted as the minimum value. These parameters are then used to compute the reliability estimator $\hat{R}$.

The true reliability R and the reliability estimator $\hat{R}$ are calculated using the robust MM-estimation method and the non-parametric kernel estimation method. Specifically, given values for the shape parameters α_i and scale parameters k_i, these parameters are estimated using the methods above, and the resulting estimates are substituted to compute $\hat{R}$ [3], [4].

2 RELIABILITY

In the concept of globalization, in the recent decades the world has witnessed openness in many fields and the accompanying liberalization of global trade. This competition supposes entry into global markets to attract foreign investment. Meanwhile, the developing countries found themselves in an unenviable position, either trying to join the developed countries in various ways or delaying. So it will be necessary to pay attention to the issues of reliability of the industrial products of all kinds in order to be able to survive and compete in the markets. Because of the importance of reliability and its applications, researchers are directed to study the failure times and reliability of most continuous distributions with the continuation of the modern development. A group of distributions appeared which are called the compounding probability distributions which were received wide attention by many researchers for their use in many fields, especially in reliability. The compounding probability distributions are one of the failure models that investigate the performance of the work of complex systems and devices. The interest has increased in estimating the reliability function of these distributions. This is to know the operational life of a number of machines and equipment by representing them with one function and to know the efficiency of these machines then evaluate them [5], [6].

2.1 The Reliability of Pareto Distribution

The probability density functions (p.d.f.) of the Pareto distribution for both stress and strength variables can be written as: [7], [8]:

$$f(x_i,\alpha_i,k_i)=\begin{cases}\dfrac{\alpha_i k_i^{\alpha_i}}{x^{\alpha_i+1}} & x_i > k_i \;,\; k_i > 0 \\ & \alpha_i > 0 \;,\; i=1,2 \\ 0 & \text{o.w}\end{cases} \quad (1)$$

Since: α_i shape parameter and k_i scale parameter. The cumulative density function (c.d.f.) can be found:

$$F_i(x_i)=1-R_i(x_i) \quad (2)$$

$$F_i(x_i)=1-\left(\frac{k_i}{x_i}\right)^{\alpha i}, \quad (3)$$

$$i=1,2 \quad , \quad x_i \geq k_i > 0 \quad , \quad \alpha_i > 0$$

The reliability function (R) for stress and toughness is:

$$R=\int_{-\infty}^{\infty} F_2(x_2)\; f_1(x_1).dx$$

$$R=\int_{\max\{k_1,k_2\}}^{\infty}\left\{1-\left(\frac{k_2}{x_2}\right)^{\alpha_2}\right\} \cdot \left\{\frac{\alpha_1 k_1^{\alpha_1}}{x_1^{\alpha_1+1}}\right\} dx \quad (4)$$

$$R=1-\frac{\alpha_1}{\alpha_1+\alpha_2}\left\{\frac{k_2}{\max\{k_1,k_2\}}\right\}^{\alpha_2}\left\{\frac{k_1}{\max\{k_1,k_2\}}\right\}^{\alpha_1}. \quad (5)$$

If the random variables of stress and strength have the same minimum, i.e. k1 = k2, then the reliability (R) in this case is [8], [9]:

$$R=\frac{\alpha_2}{\alpha_1+\alpha_2}. \quad (6)$$

2.2 Estimation Methods

2.2.1 The Robust M.M. Estimation Methods Method

This method is called Robust White. The idea behind this method is to minimize the sum of squared errors to the smallest possible extent, [5], [10], [11]:

$$\min_{a,b}\sum_{i=1}^{n}[Y_i-\beta_0-\beta_1 X_i]^2. \quad (7)$$

What the MM-estimate method uses is to minimize the following:

- P is a convex and symmetric function. To obtain robust properties and minimize the above (the sum of squared errors), we use partial derivatives of the coefficients and set them equal to zero.

$$\left.\begin{aligned}\sum_{i=1}^{n}\theta(Y_i-\beta_0-\beta_1 X_i)=0\\ \sum_{i=1}^{n}\theta(Y_i-\beta_0-\beta_1 X_i)X_i=0\end{aligned}\right\} . \quad (8)$$

- θ is the result of the partial derivative of the above function for the parameters. To obtain the solution to (8), several methods can be used, including mathematical and numerical methods such as the Newton-Raphson method, or by relying on the iteratively weighted partial least squares (IWLS) method as follows:

$$\hat{\beta}_{Rob}=\begin{bmatrix}\beta_0\\ \beta_1\end{bmatrix}=(X^T W X)^{-1}X^T W Y.$$

Note that W represents a diagonal weight matrix, and its components can be found as follows:

$$W_{ij}=\frac{\theta\,(Y_i-\beta_0^{*}-\beta_1^{*}X_i)}{Y_i-\beta_0^{\circ}-\beta_1^{\circ}X_i}, \quad (9)$$

weights through formula (9) to obtain new values for the first iteration. And then the values are in advance in the calculation and continue to repeat until we get the values of bw very close to each other in successive frequencies are stopped and the last estimate of the formula (10) is the estimate of MM [12], [13].

Since the MM estimator has a constant property, (7) can be expressed as follows. Therefore, the formula for the equation shown in (8) is as follows:

$$\left.\begin{array}{l}\sum_{i=1}^{n}\theta\left(\frac{Y_i-\beta_0-\beta_1X_i}{\hat{\delta}}\right)=0\\ \sum_{i=1}^{n}\theta\left(\frac{Y_i-\beta_0-\beta_1X_i}{\hat{\delta}}\right)X_i=0\end{array}\right\}. \quad (10)$$

Therefore, the weight matrix functions W are written as in (12) as follows:

$$W_{ij}=\frac{\theta\left(\frac{Y_i-\beta_0^{*}-\beta_1^{*}X_i}{\hat{\delta}}\right)}{\left(\frac{Y_i-\beta_0^{\circ}-\beta_1^{\circ}X_i}{\hat{\delta}}\right)}. \quad (11)$$

Estimating formulas for standard deviation values that are pre-determined before the iteration begins using one of the following formulas:

$$\begin{array}{c}1)\ \hat{\delta}=2.1\ \mathrm{Med}\ |u_i|\\ 2)\ \hat{\delta}=\frac{1}{n-4}\sum_{i=3}^{n-2}|u_i|\\ 3)\ \hat{\delta}=1.48\ \mathrm{Med}\ |u_i|\\ 4)\ \mathrm{Med}\ |u_i|/0.6745\end{array}. \quad (12)$$

where: $u_i = Y_i - \beta_0 - \beta_1 X_i$,

Where u_i represents the errors, and represents the ordered errors calculated as follows: As a result of the increasing interest in this method (estimator) over the past decades, many interested parties and researchers have used the method of robust estimators such as (Rews, Hinnich, Hameich, and Talwar) to propose many functions. This is to produce an insensitive and ineffective estimator. Some of these functions can be illustrated in the following table [14], [15], Table 1.

Table 1: Some functions are proposed as weights for fortified estimates.

Weight function	P(e)	W(e)	ψ(e)	Range
A	A^2[1-cos(e/A)]	(u/A)-1 sin(e/A)	A sin (e/A)	|e|≤Aπ
	$2A^2$	0	0	|e|>Aπ
B	$(B^2/2)[1-(e/B)^2]^3$	$[1-(e/B)^2]^2$	$u\,[1-(e/B)^2]^2$	|e|≤B
	$B^2/2$	0	0	|e|>B
T	$e^2/2$	1	u	|e|≤T
	$T^2/2$	0	0	|e|>T
H	$e^2/2$	1	u	|e|≤H
	H	H(e)-1	Sin (e)H	|e|>H

- e: Is the error to be underestimated and in our search, this is a model error in (14);
- P(e): Function in terms of variable e;
- ψ (e): A weight function designed to reduce error;
- W (e): The weight matrix used to obtain the estimates has the least error.

The constants specified in the "Range" column are used to adjust the efficiency of the resulting capacities and take the values shown in the following.

The constants specified in the “Range” column are used to adjust the efficiency of the resulting amplitudes and take the values shown in the following Table 2.

Table 2: Matrix of weights used to obtain estimates has the least error.

Weight function	T	A	B	H
Fixed parts	2.795	1.339	4.685	1.345

The MM distribution is a normal distribution of mean and variance matrix according to the following formula:

$$\mathrm{Var}-\mathrm{Cov}(\hat{\beta}_{Rob})=\sigma_{Rob}^2\,(X^TX)^{-1}. \quad (13)$$

And to appreciate it comes as follows:

$$\sigma_{Rob}^2=\frac{E(\theta^2)}{[E(\theta')]^2}. \quad (14)$$

And to be appreciated, σ_{Rob}^2 it comes as:

$$\sigma_{Rob}^2=\frac{\frac{1}{n}\sum_{i=1}^{n}\theta^2\,(r_i)}{\left[\frac{1}{n}\sum_{i=1}^{n}\theta'\,(u_i)\right]^2}. \quad (15)$$

where σ_{Rob}^2 is a biased estimate and to make it an unbiased estimate is based on the following formula:

$$\sigma_{Rob}^2=R^2\frac{\frac{1}{n-\rho}\sum_{i=1}^{n}\theta^2\,(r_i)}{\left[\frac{1}{n}\sum_{i=1}^{n}\theta'\,(r_i)\right]^2}. \quad (16)$$

Where R represents the correction coefficient and is calculated as follows:

$$R=1+\frac{\rho\,\mathrm{Var}\,(\theta')}{n\,[E(\theta')]^2}. \quad (17)$$

After obtaining the MM estimator for the model parameters i.e:

$$\hat{Y}_i=\hat{\beta}_0+\hat{\beta}_1X_i. \quad (18)$$

The estimate for the distribution of pareto for this method is:

$$F_i(x_i) = 1 - \left(\frac{v_i}{x_i}\right)^{\alpha i}, \quad (19)$$

$$1 - F_i(x_i) = \left(\frac{v_i}{x_i}\right)^{\alpha i},$$

$$\ln(1 - F_i(x_i)) = \alpha i \ \ln v_i - \alpha i \ \ln X_i,$$

$$\hat{\alpha} i = \hat{\beta}_1 \text{ and } \hat{v}_i = \exp(\hat{\beta}_0 / \hat{\beta}_1), \quad (20)$$

obtaining the MM estimator for the reliability function $\widehat{R}_{Rob}$ through the following formula:

$$\widehat{R}_{Rob} = 1 - \frac{\hat{\alpha}_1}{\hat{\alpha}_1 + \hat{\alpha}_2} \left\{\frac{\hat{v}_2}{\max\{\hat{v}_1, \hat{v}_2\}}\right\}^{\hat{\alpha}_2} \left\{\frac{\hat{v}_1}{\max\{\hat{v}_1, \hat{v}_2\}}\right\}^{\hat{\alpha}_1}. \quad (21)$$

2.2.2 Kernel Estimation Method (K.E)

This method (kernel estimation) is a scientific method for estimation clustering, agency, death, spectrum, decoration, etc. It was proposed by Barrissen and is intended to purify data by constructing capabilities with properties that match the actual parameters. There are different types of kernel estimation [16].

$$K(x) = \frac{1}{\sqrt{2\,\pi}} \exp\left(\frac{-y^2}{2}\right) \quad 0 \le y \le 1. \quad (22)$$

Although the kernel, the task of obtaining capabilities related to the characteristics of statistical research, the researcher (SAR) reiterated that choosing the dollar is not the most important step in the estimation method, rather choosing the value-added factor (H) (bandwidth bandwidth) is the most important, that is, choosing the value-added factor (1), the same requires choosing the value-added factor (h), as this factor greatly affects the bias and variance, and increasing the value-added factor leads to increased bias and reduced shrinkage and vice versa, and as a result, the prevention of value-added is determined at the degree of smoothing the curve. estimated and approaching the true curve, so the following conditions must be verified ($h = 0$; $nh = \infty$).

The value-added factor of the kernel can be fixed or change. The firmware is the use of the prime parameter along the real line used to provide the operating function and is estimated according to one of the posterior estimation methods. Ultimately it means using different values of the band and the real line in estimating the deterministic function in some cases where it becomes useful to use the fixed initial parameter and estimate that modify the appropriate sub parameters.

2.2.3 Fixed Bandwidth Kernel Estimation Method (K.F)

Suppose that T1, T2,..., TN is a sample of independent and vulnerable excellent distribution with the prosperous function and knows as follows:

$$\hat{f}_{(K.F)}(t) = \frac{1}{hn} \sum_{i=1}^{n} K\left(\frac{T_i - t}{h}\right), \quad (23)$$

$i = 1,2,\ldots n$

- K: The Kernel and the calculator of the equation of the No. (23), a specific and continuous and similar persistent functionality, has many labels, including a function (weight, format, essential) and in the following conditions [17], [18]:

$$\int_{-\infty}^{\infty} K(x)dx = 1,$$
$$\int_{-\infty}^{\infty} xK(x)dx = 0, \quad (24)$$
$$\int_{-\infty}^{\infty} x^2 K(x)dx > 0,$$

- T: represents the time of the estimation of the function of the functionality;
- n: The size of the sample is a predator (Bandwidth), fixed and estimated by the following steps;
- h: recognized fixed preparatory parameter $(\hat{h} = 1.06\ \hat{\delta} n^{-0.2})$.

The standard deviation of the sample and D difference between the normal natural quantile (valunt).

$$\hat{\delta} = \min\left[S, \frac{\widehat{D}}{1.349}\right], \quad (25)$$

$$\widehat{D} = X_{(0.25n)} - X_{(0.75n)}. \quad (26)$$

which we must receive the Kernel and its own, we can get the assessment of the Kernel and Kalati function:

$$\widehat{F}_{(K.f)}(t) = \frac{1}{hn} \sum_{i=1}^{n} \int_{\infty}^{t} K\left(\frac{X - T_i}{h}\right) dX. \quad (27)$$

Thus, the kernel estimate for the reliability function is given by:

$$\widehat{R}_{(K.F)}(t) = 1 - \left[\frac{1}{hn}\sum_{i=1}^{n}\int_{-\infty}^{t} K\left(\frac{X - T_i}{h}\right) dx\right]. \quad (28)$$

That: RK.FT Recruitment of the Proper Museum for Kernel with the Pre-Framework Practice. After the estimate of the function of the Authority by the distance that is the vulnerability of the victory, and we must compare them through the results and find out what is the perfect way or better in terms of practical application. Here they will be beaded by the modalities of estimate of the estimate, and the accuracy of the appreciation methods The accuracy of the appropriate nuclear standard: The comparison measurement for the best way:

Average error box (MSE): Where the scale is a divortiality measure of comparison between the following as the next formula

$$MSE\left(\widehat{R}_{(K.F)}(t)\right) = \frac{1}{r}\sum_{i=1}^{r}\left(\widehat{R}_{(t)} - R_{(t)}\right)^2. \quad (29)$$

Where r: represents the number of replicates per experiment.

3 STAGES OF SIMULATION

The application simulation software (R) was written and implemented on the electronic calculator, and the following steps are the steps of the simulation algorithm.

The first stage: This is the main stage upon which the steps and procedures of the program depend.

First, different default values were selected for the parameters $((\alpha_i, k_i))$. Sixteen cases were identified in Table 3.

Table 3: Represents the default values of the parameter.

Model	α	K
1	0.6	0.8
2	1.2	1.7

Second: Two sizes were selected for different samples: n=10, 30

The second stage: At this stage random observations (data) are generated by the reverse conversion method and according to the parameterized rally distribution as follows [19], [20]:

First: Generate random numbers Ui tracking the regular distribution within the period (0, t)

$$U_i \sim U_{(0,1)} \quad , i = 1,2, \dots, n.$$

It represents a continuouus random variable genearated using the electronic calculator acording to the following formula:

$$U = RND(1)$$

Second: The conversion of the data generated from step (1), which follows the regular distribution to the distribution of the railly by a parameters using the distribution function of the ralily distribution and according to the inverse method of conversion:

$$F_i(x_i) = 1 - \left(\frac{k_i}{y_i}\right)^{\alpha_i},$$

$$\text{Then } U = 1 - \left(\frac{k_i}{y_i}\right)^{\alpha_i}.$$

The function cumulative probability function for the distribution of a rally with a parameter, which we conclude: $\left(\frac{k_i}{y_i}\right)^{\alpha_i} = 1 - U$

Taking into consideration the logarithm of the parties we get:

$$y_i = \exp\left[\frac{\alpha_i \ln k_i - \ln(1-u)}{\alpha_i}\right],$$

third level: At this stage, parameters are estimated for the distribution of a parameterized rally using one of the following methods:

Method (Determined MM): In this method, the parameters of the measurement parameters is obtained and according to the formula (9), the parameter of the measurement parameters is obtained.

The fourth stage: At this stage, the reliability function is estimation as one of the concrete and non-scientific methods according to the following estimation methods.

Level five: Repeat this process for 1000 times.

Sixth stage: At this stage, it is possible to compare the quantities obtained for the distribution of the rally with the parameters and its reliability function using the following statistical criteria Average error boxes (MSE) for the reliability function.

4 DISCUSS SIMULATION RESULTS

In this section, the results of simulation experiments will be presented and analyzed to estimate the reliability function according to the solid and non-scientific methods described in the theoretical side of this research. The results shown in the tables will be analyzed in sequence and as follows:

First: To estimate the parameter for the distribution of a parameterized rally by using one of the robust methods, the sample sizes used and the different cases of the initial values shown in Table 4 and to test the number of replicates 1000 described in the tables above.

Second: The results of the statistical standard Mean error squares (MSE) for the estimated parameters of the distributed rally of all the methods and the different sample sizes and all the cases of the initial default values and the experiment of the number of replicates 1000 as shown in the tables below - Tables 4 and 5.

Thirdly, the relability function is estimation by means of the methods and the number of replicates. The results are presented in tables (4 and 5).

The results of Tables 4-5 showed that the MM-Method is the best method of estimation because it possesses the lowest MSE and percentages according to its preference sequence.

Table 4: Represents the mean square error values at a sample size of (10) and the parameter values ($\alpha = 0.6, K = 0.8$).

n	x	Rel (R(t))	MM-E	K-E
10	0.0452	0.9932	0.9903	0.9993
	0.1028	0.9853	0.9840	0.9922
	0.1281	0.9701	0.9758	0.9864
	0.2892	0.9632	0.9633	0.9705
	0.3192	0.9588	0.9536	0.9657
	0.4475	0.9532	0.9518	0.9605
	0.5111	0.9465	0.9474	0.9573
	0.6654	0.9398	0.9345	0.9443
	0.7980	0.9287	0.9265	0.9329
	0.9043	0.9150	0.9143	0.9275
MSE			0.0041	0.0048

Table 5: Represents the mean square error values at a sample size of (30) and the parameter values ($\alpha = 0.6, K = 0.8$).

n	x	Rel (R(t))	MM-E	K-E
30	0.0482	0.9902	0.9932	0.9962
	0.1022	0.9854	0.9860	0.9870
	0.2392	0.9653	0.9648	0.9731
	0.3694	0.9267	0.9318	0.9469
	0.4403	0.9032	0.9140	0.9288
	0.5930	0.8946	0.9027	0.9119
	0.6823	0.8794	0.8756	0.8947
	0.7382	0.8370	0.8445	0.8608
	0.8293	0.8195	0.8200	0.8325
	0.9011	0.7788	0.8032	0.8159
MSE			0.0036	0.0041

The results in the Tables 4,5 and 6 in the case of the assumed parameters (0.6 and 0.8) and the size of the sample (10,30 and 60) in the preferred views of the MM (Robust) method were indicated by the mean error value (MSE) of the table above as the lowest line rate.

Table 6: Represents the mean square error values at a sample size of (60) and the parameter values ($\alpha = 0.6, K = 0.8$).

n	x	Rel (R(t))	MM-E	K-E
60	0.0366	0.9254	0.9931	0.9859
	0.0978	0.9037	0.9472	0.9375
	0.1368	0.8945	0.9162	0.9047
	0.2658	0.8174	0.8936	0.8994
	0.3266	0.8004	0.8273	0.8583
	0.4002	0.7837	0.7829	0.8026
	0.5894	0.7801	0.7284	0.7482
	0.6728	0.6823	0.6296	0.7037
	0.8032	0.6173	0.5782	0.6285
	0.9467	0.5897	0.4829	0.5583
MSE			0.0028	0.0037

Table 7 represents the mean square error values at a sample size of (10) and the parameter values ($\alpha = 1.2, K = 1.7$).

n	x	Rel (R(t))	MM-E	K-E
10	0.2030	0.95145	0.9556	0.92
	0.2316	0.9427	0.9422	0.9198
	0.3679	0.9341	0.9346	0.9108
	0.4350	0.9255	0.9254	0.9016
	0.4957	0.9169	0.9162	0.9124
	0.5545	0.9084	0.9088	0.9054
	0.6636	0.9000	0.9006	0.8924
	0.6757	0.8916	0.8914	0.8804
	0.6836	0.8832	0.8836	0.8586
	0.7437	0.8749	0.8744	0.8724
MSE			0.0172	0.0183

Table 8: Represents the mean square error values at a sample size of (30) and the parameter values ($\alpha = 1.2, K = 1.7$).

n	X	Rel (R(t))	MM-E	K-E
30	0.1984	0.9040	0.9038	0.9199
	0.2504	0.8874	0.8870	0.9196
	0.3888	0.8707	0.8709	0.9132
	0.4126	0.8547	0.8535	0.8908
	0.5172	0.8379	0.8371	0.8806
	0.5346	0.8222	0.8212	0.8595
	0.6488	0.8060	0.8056	0.8123
	0.7048	0.7895	0.7886	0.8063
	0.7986	0.7728	0.7716	0.7936
	0.8708	0.7563	0.7550	0.7845
MSE			0.0096	0.0147

The results in the Tables 7, 8 and 9 in the case of the assumed parameters (1.2 and 1.7) and the size of the sample (10,30 and 60) in the preferred views of the MM (Robust) method were indicated by the mean error value (MSE) of the table above as the lowest line rate.

Table 9: Represents the mean square error values at a sample size of (60) and the parameter values ($\alpha = 1.2, K = 1.7$).

n	X	Rel (R(t))	MM-E	K-E
60	0.1583	0.9157	0.8996	0.9156
	0.1958	0.8749	0.8376	0.8438
	0.2637	0.8269	0.8167	0.8367
	0.3187	0.8033	0.7945	0.8033
	0.4008	0.7835	0.7583	0.7845
	0.4793	0.7265	0.7361	0.7558
	0.5527	0.6824	0.7022	0.7255
	0.6389	0.6278	0.6835	0.7026
	0.7278	0.5828	0.6277	0.6849
	0.8160	0.5274	0.5728	0.6187
MSE			0.0074	0.0117

5 CONCLUSIONS

The results demonstrate that the robust MM method consistently outperforms the non-parametric kernel estimation method, as indicated by the mean square error (MSE) analysis. The observed decrease in MSE values highlights the method's robustness in handling contaminated observations and its ability to accurately capture the underlying phenomenon. Moreover, the convergence of estimates from different methods with increasing sample size aligns with statistical theory, confirming the reliability of the proposed estimators. Applying these methods to other probability distributions in reliability studies may further validate their effectiveness. The simulations also show that the reliability function decreases with increasing operating time, remaining within the expected range [0,1], consistent with the theoretical properties of reliability. These findings suggest that future research could focus on estimating the reliability function for the Pareto distribution using monitoring data of both types, extending the applicability of the proposed methods.

6 RECOMMENDATIONS AND SUGGESTIONS

Hybrid algorithms that combine robust and nonparametric methods could be developed to further reduce estimation error, and the study could be extended to other probability distributions commonly used in reliability analysis, such as Weibull, Exponential, and Lognormal, to evaluate the performance and potential superiority of the MM method in different contexts. The results indicate that estimation accuracy improves with larger sample sizes; therefore, researchers and engineers are encouraged to collect sufficient data to obtain more stable and reliable results. Additionally, future research could explore the use of the truncated Pareto distribution from above as a model for failure, employing simulation methods to compare the effectiveness of various estimation approaches in this scenario.

REFERENCES

[1] B. C. Hodge, "Estimating Pareto's Constant and Gini Coefficient of the Pareto Distribution," A thesis in statistics, University of Nevada, Las Vegas, 2012.

[2] C. K. Podder, "Comparison of Two Risk Functions Using the Pareto Distribution," Pak. J. Statist., vol. 20, no. 3, pp. 367–378, 2004.

[3] J. O. David, "Applied Robust Statistics," John Wiley and Sons, Inc., 2004.

[4] M. Rytgaard, "Estimation in the Pareto Distribution," Astin, Bulletin, vol. 20, no. 2, pp. 201–216, 1990.

[5] T. H. Abdulmuttaleb, H. R. Talib, and A. Hamdullah Ahmed, "Estimation of the reliability function of the Rayleigh distribution using some robust and kernel methods," J. Phys.: Conf. Ser., vol. 1963, no. 1, p. 012064, 2021.

[6] R. Marvin, "System reliability theory, models, statistical methods, and applications," 2nd ed., Walter, Shewhart & Samuel S. Wilks, Norwegian, 2004.

[7] R. Cirillo, "The Economics of Vilfredo Pareto," Frank Cass and Company, London, 2001.

[8] C. Jin and A. Elfessi, "On the Common Scale Parameter of Several Pareto Populations in Censored Samples," Commun. Statist.-Theory Meth., vol. 30, no. 3, pp. 451–462, 2001.

[9] K. S. Trivedi, "Probability and Statistics with Reliability, Queuing and Computer Science Applications," 2nd ed., John Wiley & Sons, Inc., 2002.

[10] M. Y. Hammoud, "Comparison of nonparametric kernel estimators for estimating regression functions," Master's thesis in statistics, College of Administration and Economics, University of Baghdad, 2000.

[11] H. R. Talib, H. H. Razaq, and S. A. Madhioom, "Comparison between classical and robust estimation methods for regression model parameters in the case of incomplete data," Advances in the Theory of Nonlinear Analysis and its Applications, vol. 7, no. 4, pp. 17–34, 2023.

[12] F. Peter, "Huber sense Robust MM-estimation of a Scale Parameter with Application to the Exponential Distribution," J. Am. Stat. Assoc., vol. 74, no. 36, 1979.

[13] J. Peter Huber, "Robust statistics," John Wiley and Sons, New York, 1981.

[14] H. A. Howlader and A. M. Hossain, "Bayesian survival estimation of Pareto distribution of the second kind based on failure-censored data," Comput. Statist. Data Anal., vol. 38, pp. 301–314, 2002.

[15] A. M. Sarhan and A. I. El-Gohary, "Estimations of Parameters in Pareto Reliability Model in the Presence of Masked Data," Reliab. Eng. Syst. Saf., vol. 82, pp. 75–83, 2003.

[16] B. Miladinovic, "Kernel density estimation of reliability with applications to extreme value distribution," University of South Florida, Scholar Commons, Theses and Dissertations, 2008.

[17] H. H. Razaq, H. R. Talib, and T. K. Reem, "A comparison between two methods of estimating a semi-parametric regression model in the presence of the autocorrelation problem," Advances in the Theory of Nonlinear Analysis and its Applications, vol. 7, no. 4, pp. 54–59, 2023.

[18] E. E. Afify, "Estimation of Parameters for Pareto Distribution," [Online]. Available: http://interstat.statjournals.net/YEAR/2003/articles/0302004.pdf, 2004.

[19] N. A. J. Shammari, "The Use of Simulations in Comparing the Contractual Capacities of the Weldable Measurement Parameters of the Observational Data," PhD Dissertation in Statistics, College of Management and Economics, University of Baghdad, 2008.

[20] A. M. Al-Nasser, "An Introduction to Statistical Reliability," UB Group (Ithraa Publishing and Distribution), Amman; University Book Shop - Al-Sharjah; Elmia Book Stores - Al-Khabar, 2009.

[21] A. M. Hossain and J. T. Wiliam, "Comparisons of Methods of Estimation for a Pareto Distribution of the First Kind," Commun. Statist.-Theory Meth., vol. 29, no. 4, pp. 859–878, 2000.

[22] H. J. Malik, "Estimation of the Parameters of the Pareto Distribution," Metrika, vol. 16, pp. 126–132, 1970.