Analysis of the Application of Continuous Distribution Laws in the Reliability Theory of Technical Systems: a Review Article

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Keywords: Reliability, Distribution, Failure, Operating Time, Density, Mathematical Expectation, Variance.

Abstract: In order to enhance the accuracy of reliability assessments for highly reliable and low-volume technical systems, it is essential to utilise diverse prior information obtained through reliability calculations, simulations, testing, and operational data from structurally similar systems (analogues). From a systems approach perspective, any reliability study of technical systems should be planned and conducted considering results from previous research, i.e., incorporating prior information. Given the stochastic nature of reliability, which is an inherent property of technical systems, various discrete and continuous distributions can be employed as theoretical distributions for reliability metrics. This article analyses the practical application of key continuous probability distributions in the reliability theory of technical systems. The article presents dependencies for estimating primary reliability metrics and highlights their specific applications under different conditions. The analysis is based on a systematisation of information published in scientific and technical literature, including results from model-based and experimental reliability studies, as well as operational statistical data. The provided analytical research was carried out under a project on the prototype development of a retractable rotor system for a convertiplane. It involved making design changes to the completed technical system, which required a comprehensive assessment of reliability parameter changes at initial development stages. The results obtained allow for improving the efficiency of building models and criteria for ensuring and controlling the reliability of a convertiplane, while simultaneously increasing the accuracy of reliability estimates.

1 INTRODUCTION

During the project on the development of a retractable rotor system prototype for a convertiplane, it became necessary to modify the design of the previously developed aircraft model (Figure 1). This approach requires a comprehensive assessment of changes in the reliability parameters of the entire technical system at the initial stages of development and, therefore, the selection and justification of failure models for power and electronic components based on the distribution of relevant random variables.

To describe system failures, various models can be proposed to address different reliability tasks, each accounting for the complex factors inherent in failure mechanisms.

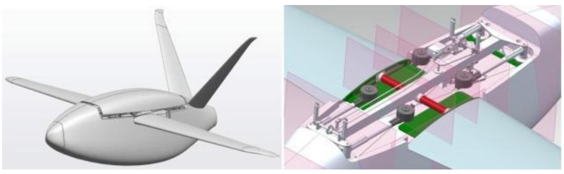

Figure 1: The technical system of aerial vehicle, the design feature of which is the presence of a retractable rotor system.

The random nature of failures during the operation of technical systems and their components allows the application of probabilistic-statistical methods. The most prevalent failure models are predicated on the distribution of pertinent random variables, such as the time to failure for nonrepairable systems and the operating time between failures for repairable systems.

The primary distributions for the time-to-failure of components include [1]: exponential; Weibull–Gnedenko; gamma; log-normal; lormal.

A review of extant scientific and technical literature in the field of technical system reliability was undertaken in order to evaluate the practical application of these distributions in the study of diverse technical systems. The analysis undertaken has enabled the selection of a suitable prior distribution for the corresponding reliability criterion or metric.

2 METHODS AND MATERIALS

This review employs a systematic methodology to analyze the application of continuous probability distributions in technical system reliability theory. The analysis is based on a critical examination of seminal and contemporary literature sourced from major scientific databases. Key distributions, including Exponential, Weibull, Normal, Lognormal, and Gamma, are evaluated for their properties in modeling failure rates, wear-out phases, and mean time between failures (MTBF).

3 EXPERIMENT AND RESULTS

3.1 Exponential (Indicative) Distribution

Despite the fact that the exponential distribution constitutes a special case of the Weibull distribution (at β=1), it is of significant independent interest, as it adequately describes the distribution of an element's operational duration during the normal operation period. The prevalence of the exponential law can be attributed not only to its diverse physical interpretations but also to its remarkable simplicity and convenience in modelling properties. The following formulae are provided for determining the probability density f(t) and the probability of failure-free operation R(t) over time tt, corresponding to this law:

$$f(t) = \lambda e^{-\lambda t}, \quad (1)$$

$$R(t) = e^{-\lambda t}. \quad (2)$$

where λ is the failure rate.

The mathematical expectation m and the standard deviation σ for the exponential distribution are expressed through the following parameter:

$$m = \sigma = \frac{1}{\lambda}. \quad (3)$$

The mean time to the first failure equals:

$$T_1 = \frac{1}{\lambda}. \quad (4)$$

The exponential distribution is frequently employed during the design stage when information regarding the reliability of the system's components is limited or entirely absent. Consequently, it is often referred to as the fundamental law of reliability [2]. A limitation of this law is the requirement that failure and restoration flows must be the simplest (possessing properties of ordinariness, stationarity, and lack of aftereffect) [3].

As asserted by [4]-[12], the exponential distribution is a reliable model for the reliability of equipment operated following the run-in period and prior to the onset of significant gradual failures, that is to say, during normal operation, when sudden failures predominate. As stated in [2], [7], the time to failure of technical systems with a large number of serially connected elements can be described by this distribution if each individual element does not substantially influence the system's failure. In such instances, if the failures of the serially connected elements follow an exponential distribution, it can be deduced that the system's failures will also adhere to this law, and its failure rate will equal the sum of the failure rates of the elements. It is important to note that systems comprising elements connected non-serially in terms of reliability will not exhibit an exponential distribution, even if the failure-free probabilities of their components are exponential [3]. Given that each system element is itself a subsystem comprising multiple (often numerous) components, the total failure rate of the system depends solely on the number of faulty elements, and the repair time for each failed element follows an exponential distribution. The subsystem's failure is equated to the failure of one of its elements, which is replaced with a new one upon restoration. The subsystem's net operational time ensures that its failure flow will asymptotically approach a Poisson process due to the Khinchin limit theorem. Consequently, the time interval between consecutive failures will also follow an exponential distribution [13], [14].

The exponential law is applicable to complex technical systems in which numerous destructive processes with varying rates occur simultaneously. However, as the heterogeneity of process rates decreases, the distribution approaches a normal one. Conversely, when homogeneous destructive processes dominate, the distribution corresponds precisely to the normal distribution [11].

As posited by the authors of [2], [4], [6], [15], in circumstances pertaining to the maintenance of intricate systems, the restoration flow, if uncomplicated, is best characterized by the exponential law. This law serves to elucidate the restoration intensity, maintenance labor intensity, and failure rectification. In the context of queuing theory, the exponential law provides a reliable framework for modeling the intervals between equipment arrivals for repair [3, SEVERTSEV].

In the context of standard operation, sudden failures are attributed exclusively to external impacts, thereby negating the possibility of a causal influence by replacing an existing element with a new one. Consequently, for systems governed by the exponential distribution of time to failure, preventive measures such as preemptive replacements or periodic repairs are rendered moot [16].

As indicated in [2], [3], the exponential law has been observed to provide a reliable approximation of the failure-free probabilities of a wide range of technical objects, including components of radio-electronic equipment, electrical and electronic devices, and hardware-software complexes. This approximation is particularly relevant when considering the physical nature of sudden failures.

Nevertheless, despite its simplicity and universality, the exponential law has several limitations. In particular, works [3], [13], [17] call into question the validity of applying the exponential distribution to long-term operational systems over extended time intervals for the following reasons:

The distribution's "memoryless" property introduces a significant drawback, contradicting natural physical interpretations. This property implies the absence of ageing, meaning a technical object does not degrade over time or, after a certain operational period, exhibits the same failure distribution as a new object. This property is invalid for many technical objects, especially over extended periods [3], [13], [17].

In [3], it is asserted that the exponential distribution is inapplicable to complex technical systems, as the non-simultaneous operation of components and the presence of failure aftereffects result in a non-constant system failure rate, even if the failure rates of individual components remain constant. Consequently, the application of the exponential distribution to analyse the reliability of real long-term operational systems is invalid, as the model assumptions do not align with the physical processes within such systems.

It is therefore vital that robust justification is provided for the application of the exponential distribution, as with any other such model. Nevertheless, its ubiquity endures for the reasons outlined below:

It is noteworthy that the system is both simple and reliant upon a single parameter λ. This, in conjunction with the memoryless property, facilitates the development of analytical solutions to a multitude of reliability problems.

It has been demonstrated that the time to failure of complex, highly reliable, repairable systems can be described by the exponential distribution under specific conditions (e.g., negligible material "ageing" effects).

The utilisation of the exponential law is deemed permissible when the procurement of conservative reliability estimates, that is to say, lower-bound reliability assessments, is permitted under specific conditions.

3.2 Normal Distribution Law (Gaussian Distribution)

The extensive corpus of theoretical research on the normal distribution law, in conjunction with its relatively elementary mathematical properties, renders it highly attractive and convenient for application in reliability theory. In instances where empirical data deviates from the normal distribution, the following approaches may be employed for its judicious use:

The utilisation of this approach as a preliminary estimation frequently yields adequately precise outcomes.

It is imperative to ascertain a transformation of the studied random variable, designated as ξ, that modifies the original 'non-normal' distribution into a normal one [14].

A salient property of this law is its capacity for self-reproduction, which stipulates that the aggregate of any number of normally distributed random variables also adheres to a normal distribution.

The probability density function of this law is defined as:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-m)^2}{2\sigma^2}}. \quad (5)$$

where m is the mean and σ is the standard deviation.

The reliability function P(t)P(t) for the normal distribution is calculated using:

$$R(t) = \int_{t}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx = = 0{,}5 - \Phi_0\left(\frac{t-m}{\sigma}\right), \quad (6)$$

where $\Phi_0(t) = \frac{1}{\sqrt{2\pi}} \int_{0}^{t} e^{-\frac{x^2}{2}} dx$ is the Laplace function, the values of which are tabulated.

The mean time to failure is $T_{cp} = m$, and the relationship between the time to first failure $\underline{T_1}$ and T_{cp} is expressed by:

$$\underline{T_1} = T_{cp} + \frac{\sigma\sqrt{\frac{2}{\pi}}}{\left[1+\Phi\left(\frac{T_{cp}}{\sigma\sqrt{2}}\right)\right]} e^{-\frac{T_{cp}^2}{2\sigma^2}}. \quad (7)$$

The failure rate for the normal distribution is an increasing function defined as:

$$\lambda(t) = \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{\left(t-T_{cp}\right)^2}{2\sigma^2}}}{\sigma\left[1-\Phi\left(\frac{t-T_{cp}}{\sigma\sqrt{2}}\right)\right]}, \quad (8)$$

where Φ is the probability integral of the $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-x^2} dx$ form.

The normal law is employed to delineate the reliability of technical objects during the aging period [2], [4], [6], [18]. According to several sources [6]–[9], [19]–[21], its applicability is indicated when failures occur gradually due to directed physico-chemical changes caused by wear (ageing), provided the coefficient of variation $\upsilon \leq 0{,}3 \div 0{,}4$ [7], [22]. Under stable operating conditions, this distribution effectively models the mean and gamma-percentile resource [7] as well as the operational time of machinery until the first major overhaul [4].

It is important to note that the normal distribution of time-to-failure arises from the homogeneity of technical object quality, constant average wear rate, and cumulative wear effects, which interweave over extended periods before failures manifest [16].

A salient feature of the normal law is that when σ is small relative to the mean time to failure m, the density values remain close to zero over a significant time interval. This phenomenon can be attributed to the low probability of failure within this interval, which can be utilised to schedule preventive replacements (or repairs) at low wear levels, thereby minimising the likelihood of failure between maintenance actions [16]. Conversely, the exponential distribution, with its density peak at t = 0, exhibits the majority of failures during the initial operational phase.

Statistical analyses [2], [11] of test and operational data for mechanical components and metal structures subject to intense wear, ageing, and fatigue phenomena demonstrate that strength and load distributions conform to the normal law. Furthermore, the combination of exponential and normal laws has been observed in systems where one or more degradation processes dominate, such as in hydraulic power systems and gear pumps, where the normal law provides an accurate description of the time between failures [11]. Furthermore, the Gaussian distribution is observed to govern random variables such as measurement and manufacturing errors [16].

3.3 Log-Normal Distribution

The log-normal distribution of a random variable, designated here by the letter "ξ", is established when its logarithm conforms to a normal distribution. The values of a log-normally distributed random variable are influenced by a large number of mutually independent factors, each of which contributes uniformly insignificantly and with equal probability in sign. In contrast to the normal distribution, the sequential nature of these factors' impact is such that the random increment caused by each subsequent factor is proportional to the value already attained by the variable at that moment. The log-normal distribution finds wide application in reliability theory, particularly in the analysis of empirical data on subjects such as the fatigue durability of metals, the long-term strength of materials, and the time-to-failure of electronic components, among others [LRS].

The probability density function is described by the relationship:

$$f(t) = \frac{1}{st\sqrt{2\pi}} e^{-\frac{(ln ln\, t - \mu)^2}{2s^2}}, \quad (9)$$

where $\mu = \frac{1}{n}\sum_{i=1}^{n} ln\, ln\, t_i$ and $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (ln\, ln\, t_i - \mu)^2}$ are parameters estimated from nn failure test results.

For the log-normal distribution, the reliability function is expressed as:

$$R(t) = \frac{1}{\sqrt{2\pi}} \int_{\frac{\left(\frac{t}{\mu}\right)}{s}}^{\infty} e^{-\frac{t^2}{2}} dt. \quad (10)$$

The mean time to failure and standard deviation are determined by:

$$T_{cp} = m = e^{\left(\mu + \frac{s^2}{2}\right)}, \quad (11)$$

$$\sigma = \sqrt{e^{2\mu + s^2}\left(e^{s^2} - 1\right)}. \quad (12)$$

The failure rate for the log-normal distribution is given by [23]:

$$\lambda(t) = \frac{0{,}4343 e^{-\frac{(lglg\, t - \mu)^2}{2s^2}}}{st\sqrt{2\pi}\Phi\left(\frac{\mu - lglg\, t}{s}\right)}. \quad (13)$$

The log-normal distribution is well-suited to positive-valued quantities, providing enhanced precision in comparison to the normal distribution in such scenarios [2]. The log-normal distribution is a model for the time-to-failure behaviour of objects that exhibit "strengthening" over time. This phenomenon of "strengthening" leads to a gradual reduction in wear rate. Consequently, a prerequisite for the application of the log-normal distribution is the verification of the "strengthening" property exhibited by the technical objects under study. This can be achieved through analysis of wear processes and, where possible, empirical wear patterns [16].

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This distribution is also used to model:

- Recovery processes;
- Product durability during ageing, where wear increment is proportional to the instantaneous wear value [7], [14], [19];
- Operating time under rapid "burn-out" of unreliable components;
- Failures caused by material fatigue, such as the operational lifespan of rolling bearings, vacuum tubes, etc. [3], [7].

It is evident that the log-normal distribution provides an effective description of the time-to-failure of complex technical systems (e.g. tractors, automobiles, heavy-duty machinery) and electronic equipment [2].

3.4 Gamma Distribution

The gamma distribution has a two-parameter probability density function with a shape parameter (α>-1) and a scale parameter (β>0):

$$f(t) = \frac{t^{\alpha}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{t}{\beta}}. \quad (14)$$

where $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$ is the gamma function.

The probability of failure-free operation is determined by the formula:

$$R(t) = \int_t^{\infty} \frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{\frac{t}{\beta}} dt. \quad (15)$$

The parameter α, which characterises the asymmetry of the gamma distribution, determines the type of reliability characteristics.

The mathematical expectation (mean time to failure) and standard deviation for the gamma distribution are:

$$T_1 = m = \alpha\beta\ , \quad (16)$$

$$\sigma = \sqrt{\alpha}\beta. \quad (17)$$

The formula for failure rate is:

$$\lambda = \frac{t^{\alpha-1}\beta^{-\alpha}}{\Gamma(\alpha)\sum_{i=0}^{\alpha-1} \frac{t^i}{\beta^i \cdot i!}}. \quad (18)$$

The gamma distribution is used to describe:

- Wear-out failures;
- Failures due to cumulative damage;
- Time-to-failure of complex technical systems with redundant elements;
- Time-to-repair distribution [2], [7], [10], [16];
- Durability (service life) of certain technical objects [17].

The gamma distribution possesses several useful properties [20]:

For α<1: The failure rate decreases monotonically (burn-in period), corresponding to rapid elimination of unreliable components.

For "α>1" : The failure rate increases (wear-out period), reflecting gradual ageing and degradation.

For "α=" 1: The gamma distribution coincides with the exponential distribution, applicable during normal operation [18].

For "α>" 10: It closely approximates the normal distribution, suitable for ageing components [16], [18]. In addition, as R(t)→∞, the gamma distribution converges to the normal distribution law. Consequently, it is frequently employed to approximate single-vertex but asymmetric distributions [9], [12].

If α is a positive integer, the gamma distribution is also known as the Erlang distribution [2], [7].

For λ=1/2 and α multiples of 1/2, it coincides with the chi-squared distribution [2], [7].

The gamma distribution is a statistical model frequently employed to analyse the failure times of complex electromechanical systems during the initial operational phases or the debugging process, where early failures are observed [11], [12]. Furthermore, the reliability of systems composed of exponentially distributed elements follows a gamma distribution [11].

In redundant systems (with standby or mixed redundancy), failure time distributions adhere to a generalised gamma distribution [3], [12]. However, the presence of hidden failures and the limitations of diagnostic effectiveness in such systems necessitate detailed modelling (e.g., via Markov or semi-Markov processes) [23].

3.5 Weibull Distribution

The Weibull distribution is characterised by its capacity to model data from distributions exhibiting increasing, decreasing, or constant failure rates. The two-parameter Weibull distribution, characterised by a scale parameter, denoted here by ηη, and a shape parameter, denoted here by ββ, is a widely utilised tool in the analysis of reliability data and scenarios where the process of ageing is induced by specific stress factors (e.g. pressure, temperature).

The probability density function of the distribution is defined as follows:

$$f(t) = \beta \frac{t^{\beta-1}}{\eta^{\beta}} e^{-\left(\frac{t}{\eta}\right)^{\beta}}. \quad (19)$$

The relationships between reliability metrics are as follows:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}, \quad (20)$$

$$T_1 = \alpha\Gamma\left(1 + \frac{1}{\beta}\right), \quad (21)$$

$$\sigma = \alpha\sqrt{\Gamma(1 + 2\beta^{-1}) - \Gamma^2(1 + \beta^{-1})}, \quad (22)$$

$$\lambda(t) = \beta\alpha^{\beta} t^{\beta-1}. \quad (23)$$

This distribution is highly versatile, encompassing applications of several other distributions, albeit described by more complex formulae [4]. It is applicable for modelling:

The service life of rolling bearings, threaded connections, splined shafts, and other components subject to simultaneous wear across multiple working surfaces [4];

Time-to-failure under combined sudden and wear-out failure mechanisms [4];

Probability of failure-free operation of mechanical components during ageing or wear-out phases [3], [11], [12];

Service life of structural components (e.g., load-bearing systems, hydraulic and electric drive systems) affected by fatigue and sudden failures (with a coefficient of variation between 0.35 and 0.7);

Failure distributions during the burn-in period [2], [3], [21];

Operational lifespan of complex technical systems (e.g., mobile installations) [2], [18];

Fatigue-related failures in automotive parts, lifting machinery, and bearings [2], [3];

Mean and gamma-percentile resource under stable operational conditions [7].

In certain cases [3], [8], [10], the Weibull distribution is considered universal due to its properties:

When β=1, the distribution reduces to the exponential distribution, with a constant failure rate λ(t) that equals the inverse of the scale parameter η;

For β<1, the density and failure rate functions are decreasing (burn-in period);

For β>1, the density and failure rate functions are increasing (wear-out period);

At β=2, the λ(t) function is linear, the distribution transitions to the Rayleigh distribution with density β=3,44, the density closely approximates the normal distribution, excluding tail regions.

The Weibull distribution is recommended as a primary candidate for approximating empirical reliability data when the underlying distribution is unknown [15], owing to its flexibility. In addition, it has been demonstrated that the Weibull distribution provides a superior fit to real-world data in comparison to the exponential distribution [9].

As with the gamma distribution, the Weibull distribution is effective in approximating real-world failure patterns across all lifecycle stages (burn-in (β<1), normal operation (β=1), and wear-out (β>1)) [2], [14], [20].

In the context of complex systems comprising multiple components with gamma-distributed lifetimes (where parameters vary slightly between components), the system's failure time distribution tends to converge to the Weibull distribution [16], [20]. Systems with multiple identical or similar components under comparable operational conditions (e.g., internal combustion engines with multiple cylinders, electronic devices with capacitors and resistors) often follow the Weibull distribution if these components dominate the system's reliability [16].

From a physical perspective, the Weibull distribution provides an adequate description of the time-to-failure of electronic components, with failure defined as a parameter exceeding specified thresholds [16].

4 DISCUSSION

In conclusion, it should be noted that, in addition to the distributions discussed herein, specialised types (amounting to several dozen) as well as discrete distributions – which fall outside the scope of this article – are applied to address specific tasks. Furthermore, a variety of interrelations and transitions exist between these distributions. Despite the existence of established goodness-of-fit criteria for evaluating the alignment of theoretical and empirical distributions, the ultimate question addressed is whether there are sufficient grounds to reject the hypothesis of the chosen distribution. As the authors [7] have noted, it is important to understand that any dataset can be fitted to a multi-parametric law, even if it does not correspond to actual physical phenomena. Consequently, when selecting the type of distribution and its parameters, it is imperative to prioritise the physical factors governing the degradation process leading to failure, characterised by natural (physical) regularities, as well as the state of the production technological process, which significantly influences the form of time-to-failure distributions. The results obtained allow for improving the efficiency of modelling and establishing criteria for ensuring and controlling the reliability of a convertiplane with an automatic beam deployment system at initial development stages [24], [25].

5 CONCLUSIONS

Existing models of technical system operation, particularly the exponential model, allow for a comparative assessment of the reliability of various design solutions and the selection of the best one. However, they do not provide sufficiently accurate estimates of performance indicators. When selecting distribution laws, the relative error can be very high and even unbounded. Furthermore, the use of exponential laws leads to significant discrepancies between analytical and experimental data. Real physical processes in technical systems, when examined in detail, represent a combination (mixture) of distributions. The scientific problem of reliability analysis for repairable systems with arbitrary failure and recovery distributions is primarily a technical challenge arising from the properties of complex systems. Therefore, a future approach may involve a transition from quantitative metrics to scientifically substantiated qualitative ones.

ACKNOWLEDGMENTS

The work was carried out using a grant provided in 2024 by the Academy of Sciences of the Republic of Tatarstan for the implementation of fundamental and applied scientific work in scientific and educational organizations, enterprises and organizations of the real sector of the economy of the Republic of Tatarstan, agreement No. 14/2024-FIP dated 12/24/2024.

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