Remote Sensing-Based Numerical Methods for Water Surface Area Estimation

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Abstract Monitoring water bodies in Iraq is vital in light of increasing climatic and human pressures, especially for Lake Hamrin, a major source of drinking and irrigation water in Diyala Governorate. This research relied on Sentinel-2 L2A images from the Copernicus program for the period 2019–2025 to determine changes in the lake's surface area. The Normalized Water Index (NDWI) was used to extract the water boundaries, then convert the geographic coordinates to metric (UTM) coordinates to accurately draw the shoreline. Two numerical methods were applied to calculate the water surface area of Lake Hamrin the Shoelace and Simpson's methods. Comparing the results with reference values extracted from the Copernicus platform, it was found that the shoelace method yielded a lower error (0.7–3.1%) than the Simpson method (1.3–5.1%). This is attributed to the complexity of the coastal shape of Lake Hamrin. Shoelaces provide a more accurate representation of coastal details, while Simpson's method tends to simplify the boundaries. However, the differences between the two methods diminish when the water level is low, as in 2021–2023, as the shape of the lake becomes more regular.

1 INTRODUCTION

Iraq's natural and artificial lakes such as Tharthar, Habbaniyah, Razzaza, Hamrin, Dukan, Darbandikhan, and the Mosul Dam reservoir are vital pillars of water security, the environment, and the economy (flow storage and regulation, irrigation, hydropower, fishing, and biodiversity) [1]. Over the past two decades, strong observational evidence has accumulated that these bodies of water are subject to increasing pressures due to climate change (increasing temperatures, increased evaporation, and more frequent droughts) and human factors (upstream regulations and transboundary hydropower projects) [2]. This pressure has resulted in shrinking surface area, fluctuating water levels, and deteriorating water quality characteristics. Analyses of water budgets and climate in the Tigris and Euphrates basins show that droughts and extreme heat waves have become a central player in resource scarcity and fluctuating surface storage, with wide-ranging social and environmental impacts [3]. One of the most important of these lakes is Lake Hamrin. It is water basin that was completely submerged in 1981 with the construction of the Hamrin Dam as part of preventative measures against the flooding that plagued Iraqi cities. The lake supplies Diyala Governorate with approximately 70-80 percent of its drinking and agricultural water needs, according to the governorate's Water Resources Directorate, with a capacity of more than 2.4 billion cubic meters of water. Therefore, due to the importance of this lake and other lakes in Iraq, we need to monitor this lake with modern technologies that save time and effort and provide sufficient information upon which to base decisions, especially since the world is experiencing dangerous climate changes. One of the most important of these modern technological methods is remote sensing and the technologies associated with it [4]. At the spatial level, remote sensing imagery has provided a highly reliable time series for tracking water surface fluctuations across Iraq's large water bodies. Recent studies have observed clear seasonal and interannual shrinkage in major lakes such as Tharthar, Habbaniyah, and Hamrin, linking this to increasing climate stress and declining water releases, highlighting the sensitivity of these reservoirs to air temperature and increased evaporation. Other studies have documented the shrinkage of water surface boundaries [4], [5].

In dry and semi-dry environments such as Iraq, peer-reviewed studies have demonstrated significant shrinkages in major reservoirs and lakes (such as Razzaza and Hamrin) based on Sentinel-2, classification algorithms, and hydrological indices, supporting water management decisions under drought and heat stress. Jumaah et al. (2023) studied the decline of Lake Hamrin using Sentinel-2 satellite imagery and GIS analysis. Using the Normalized Water Index (NDWI), the researchers documented a sharp decline in the lake's surface area between 2017 and 2022, with the water cover decreasing from approximately 144 km² to less than 40 km² [6]. Ameen et al. (2025) extended this research by analyzing the hydrological dynamics of Lake Hamrin over two decades (2004–2024). Using a multi-temporal approach using GIS, they were able to measure spatial and temporal changes in both coastal morphology and water area [7]. Mousa (2022) focused on Lake Sawa in southern Iraq, presenting a comparative case study of the long-term degradation of closed lakes. Using Landsat imagery from 1985 to 2020, the study revealed a continuous decrease in surface area, attributing this to declining groundwater recharge, over-abstraction, and prolonged droughts [8].

Remote sensing is a quantitative framework for non-contact Earth surface observation. Change detection methods were established early on through systematic reviews that formed multi-temporal comparison and their spectral/statistical processing. Capacities have evolved from comparing two or more snapshots to continuous time series observations using all Landsat views. Within aquatic systems, the NDWI and MNDWI spectral indices have become the standard for extracting water bodies and tracking their expansion and contraction in different environments [9].

Sentinel-2 is a wide-band multispectral imaging mission for land and vegetation monitoring, built on a constellation of identical satellites to achieve high pass frequency. Each platform carries a 13-band MSI with 10/20/60 m spatial resolution, a 290 km path width, and a 5-day revisit time at the equator (with two satellites in operation). The S2A (2015) and S2B (2017) satellites were launched, followed by S2C (5 September 2024) to enhance operational continuity. Sentinel-2 is a part of the Copernicus program. Copernicus is the Earth observation component of the European Union Space Program, managed by the European Commission in partnership with the European Space Agency (ESA) and other European stakeholders. It provides data and operational services to support environmental management, climate change adaptation, and civil security, under a policy of free and open access to users. The program includes the Sentinel families (radar, optical, oceanographic, etc.), as well as thematic services (land, air, oceans, climate, emergencies, and security) [10].

2 MATERIAL AND METHOD

The Hamrin Dam Lake (Lake Hamrin) was designated as the research area on the Diyala River, about 120 km northeast of Baghdad, Iraq, 340-square-kilometer. It is an artificial reservoir located between 34° 00′ 30″ to 34° 21′ 30″ N and 44° 52′ 00″ to 45° 12′ 00″ E, created after the establishment of Hamrin Dam in June 1981 Figure 1. The principal objective of Hamrin Dam's construction was to regulate the yearly flooding of the Diyala River; it furthermore functions as a vital irrigation supply and generates electrical power for Diyala Governorate.

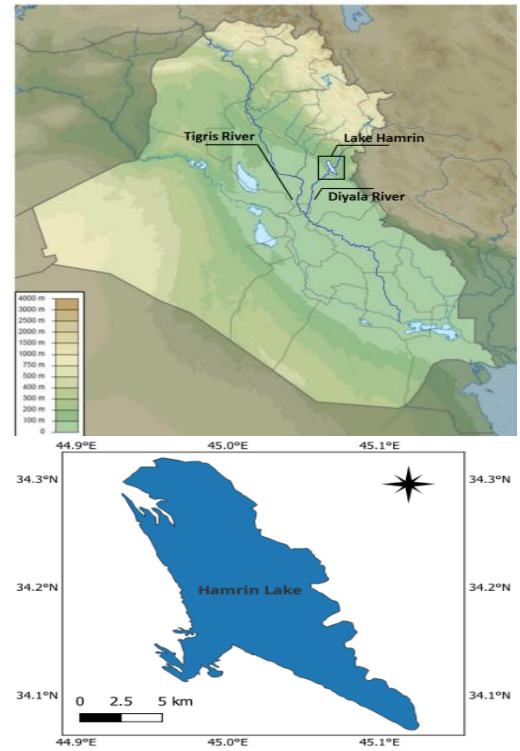


Figure 1: Geographical location of Lake Hamrin within the map of Iraq [6].

Most methods for calculating lake area from satellite images rely on spectral separation of image classes. Then the surface area of water in that lake is calculated using several different mathematical methods. This study introduced numerical method to estimate the water surface area of the lake Hamrin by Shoelace method and Simpson's. The results of these methods were compared with the calculation of the surface area of the lake water calculated by the Copernicus website [7], [9]. The main steps of this work are explained as Figure 2.

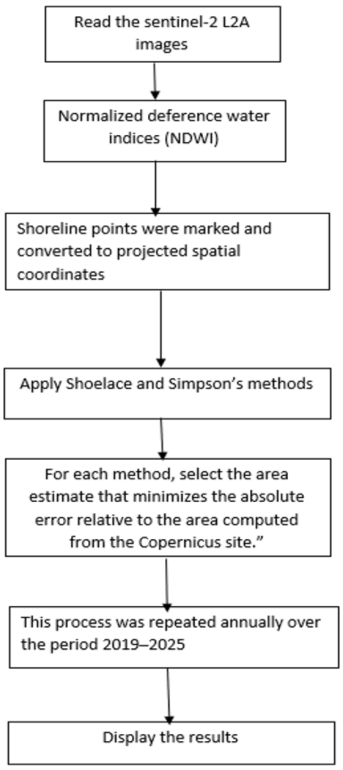


Figure 2: Block diagram for methodology of the work.

The Shoelace algorithm is the standard method for calculating the area of any polygon whose vertices are known in order (the boundary of the lake after being converted to a polygon). It works directly on plane coordinates. We arrange the boundary points x_i, y_i around the circumference (clockwise/ counterclockwise), then add the two "directions" and take half the absolute difference between them [11]. This gives a signed area: positive when arranged counterclockwise and negative when arranged clockwise. For a simple polygon has n vertices the formula of shoelace is given as follow (1):

$$A = 1/2 \sum_{i=1}^{n} |x_i * y_{i+1} + x_{i+1} * y_i|. \quad (1)$$

A is area of polygon.

Simpson's Rule is used to approximate definite integrals and relies on approximating the function using a parabola (a second-degree polynomial) between the division points. The interval is divided into an even number of subintervals [12]. Simpson's Rule is yet another method of numerical integration, given by the (2):

$$\mathrm{J} = \int_a^b f(x) = \frac{\mathrm{h}}{3} [\mathrm{fo} + \mathrm{fn} + 4\sum_{i=odd}^{n-1} f(i) + 2\sum_{i=even}^{n-2} f(i)]. \quad (2)$$

Where: $h = \frac{b-a}{n}$.

Sentinel-2 is a European mission that utilizes wide-swath, high-resolution, multi-spectral imaging. Dedicated to Europe's Copernicus program, the mission supports operational applications primarily for land services, including the monitoring of vegetation, soil and water cover, as well as the observation of inland waterways and coastal areas. The twin satellites S-2B and S-2C fly in the same orbit but are phased at 180°, and the full mission specification is designed to provide a high revisit frequency of 5 days at the Equator. In addition, satellite S-2A is temporarily operating at position 36° away from S-2B. Sentinel-2 L2A refers to the Level-2A data processing for the Sentinel-2 satellite, which is part of the Copernicus program by the European Space Agency (ESA).

Sentinel-2 L2A images from the European Copernicus site were used to study the deterioration occurring in Lake Hamrin for a period from 2019 to 2025, where seven images were obtained to cover this period over five years [10], [13]. Locations on the Earth's surface are typically recorded using geographic coordinates, represented by longitude and latitude in degrees. However, this system is impractical when it comes to linear measurements or calculating distances and areas, as it relies on an irregular elliptical surface. To overcome this, geographic coordinates are converted to a projected coordinate system, such as UTM (Universal Transverse Mercator) or Gaussian–Krüger. These coordinates are then converted to metric values (Easing, Northing) on a flat surface. The conversion process is performed using mathematical projection equations that take into account the Earth's elliptical shape. Points are then projected onto a two-dimensional plane using parameters such as the polar radius, central longitude, and scale factor. For example, in the UTM system, the globe is divided into longitudinal zones of 6 degrees width, and the Transverse Mercator projection is applied to each zone to minimize distortions. The result is a metric coordinate system that allows for the calculation of distances and angles with high accuracy, making it essential for applications in geographic information systems (GIS), remote sensing, and surveying. For this purpose, often use GIS tools such as ArcGIS or QGIS [14], [7].

The Normalized Difference Water Index (NDWI) is one of the most important spectral indices derived from satellite imagery. It is widely used to detect surface water bodies (such as rivers, lakes, and reservoirs) and monitor changes in their spatial and temporal distribution. NDWI relies on the spectral contrast between the high reflectivity of water in the green band and its low reflectivity in the near-infrared band (NIR) [15]. The mathematical equation for calculating NDWI is:

$$NDWI = (Green - NIR) / (Green + NIR). \quad (3)$$

Green: Green band reflectivity (usually Landsat band 2 or 3, depending on the sensor).

NIR. Near-infrared band reflectivity (usually Landsat band 4 or 5).

The Sentinel-2 L2A (Level-2A) data from the Copernicus program provide surface reflectance after atmospheric correction, making it suitable for accurate spectral analyses such as extracting water cover indices. The Normalized Difference Water Index (NDWI) from Sentinel-2 L2A is calculated using two channels [16]:

$$NDVI = (B3 - B8)/(B3 + B8). \quad (4)$$

- B3 (Green, 560 nm, 10 m spatial resolution);
- B8 (NIR, 842 nm, 10 m spatial resolution).

We will choose large number of points around the lake shore so that they cover all the details of the shore of lake, including dents, cracks, and bends, so that we get as close as possible from the actual area of the lake's water surface also for more accurate.

A sufficient number of points was selected to cover all details of the lake shoreline, including indentations, curvatures, and irregularities, in order to apply both Simpson’s Rule and the Shoelace Formula. This approach ensures achieving high accuracy in calculating the lake’s surface area using these methods. Figure 2 illustrates steps the methodology of the work.

3 RESULTS AND DISCUTIONS

The Hamrin Lake is an artificial reservoir created in 1981 after the construction of the Hamrin Dam on the Diyala River in Diyala Governorate, northeastern Iraq. Its variable surface area is approximately 300–400 km², depending on the water storage level. Figure 3 shows two sets of images taken from the Copernicus site during the period from 2019-2025 in the month of July, which represents the drought month in Iraq. A sufficient number of points were marked around the coast of Lake Hamrin, shown in white. After that, the coordinates of these points were recorded and converted from the Geographic Coordinates to the metric coordinates (UTM). The shoelace method and the Simpson method were applied, and the results were included in Table 1.

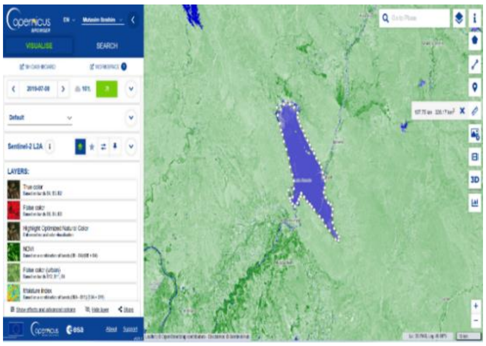
Table 1: The calculated area of water surface of Lake Hamrin in Km^2 using three methods for the period from 2019 – 2025.

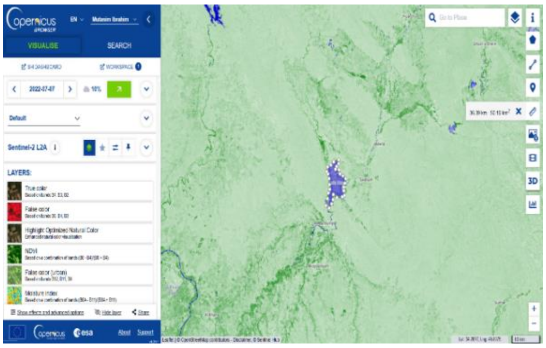
Year	Copernicus	Shoelace Method	Simpson’s method
2019	107.75	105.2	102.3
2020	77.17	76.11	75.21
2021	38.81	37.61	37.32
2022	36.39	36.13	35.84
2023	49.58	49.12	48.93
2024	81.71	80.51	79.73
2025	101.38	99.34	98.71

In Table 1, the relative error is defined as the ratio of the absolute error to the reference (true) value. The Percentage Relative Error (PRE) is expressed as follows:

$$PRE = (|Numerical\ Value - Copernicus\ Value|)/(Copernicus\ Value) * 100\% \quad (5)$$

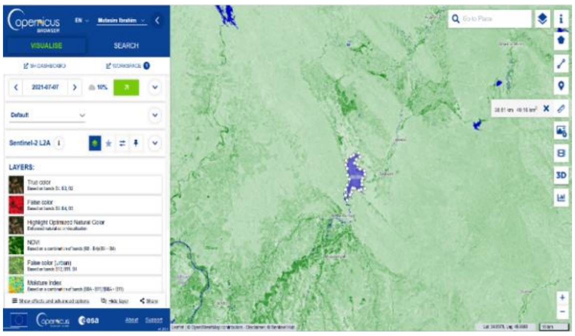
Here, the numerical value denotes the surface area calculated by each method. By applying (5) [17], [18], the percentage relative error (PRE) was obtained for both methods over the study period 2019–2025, thereby providing a quantitative assessment of their accuracy relative to the Copernicus-derived values. The comparison between Copernicus-derived water surface areas and the results of the Shoelace and Simpson's methods demonstrates consistent accuracy across the study period (Fig. 3). The Shoelace method produced values with lower absolute and relative errors (0.7–3.1%) compared to Simpson's method (1.3–5.1%) (Fig. 4). This outcome is logically consistent with the complex geometry of Lake Hamrin's shoreline, which is characterized by numerous indentations and irregularities. Since the Shoelace formula directly utilizes the spatial coordinates of all vertices along the lake boundary, it is able to capture fine shoreline details more accurately than Simpson's numerical approximation. Therefore, the Shoelace method is considered more reliable for estimating the water surface area of Lake Hamrin.

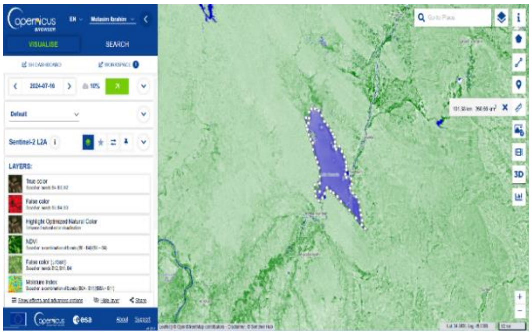

Lake Hamrin 2019


Lake Hamrin 2022


Lake Hamrin 2020


Lake Hamrin 2023


Lake Hamrin 2021


Lake Hamrin 2024

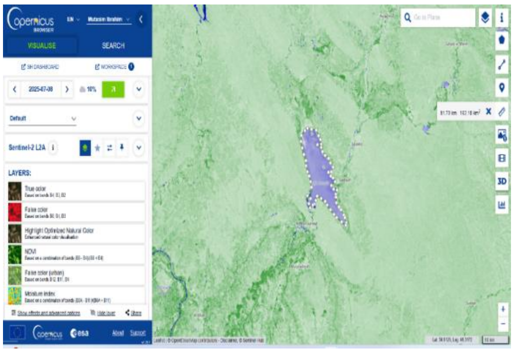


Figure 3: Shoreline of Lake Hamrin marked with white points during 2019–2025.

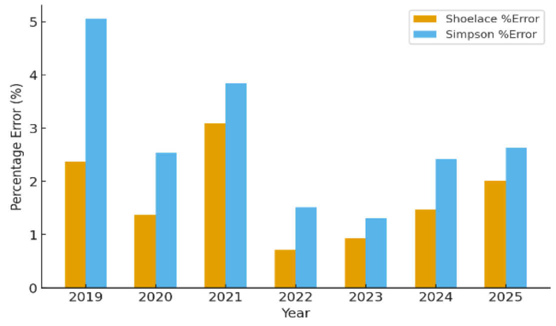


Figure 4: Percentage relative error of shoelace verses simpson's methods (2019–2025).

It can be observed from the Table 1 that during the years 2021, 2022, and 2023 the difference between Simpson's and the Shoelace methods is relatively small. This reduction in discrepancy is primarily attributed to the significant decline in the water surface area of the lake during these years. When the water extent decreases, the shoreline becomes less complex with fewer indentations and irregularities, which reduces the geometric sensitivity of the applied methods. Consequently, both Simpson's numerical integration and the Shoelace polygon-based calculation produce closely aligned results under such conditions. Simpson's method relies on the numerical integration method by dividing the shape into parabolic segments. It is suitable for relatively regular shapes or when the region's boundaries are close to smooth curves. The Shoelace method is a straightforward algebraic formula for calculating the area of a polygon from its vertices' coordinates (x, y). It gives more accurate results for complex and irregular shapes. It relies solely on point coordinates and can accurately represent coastal details if the number of points is large. It does not require integration or approximation assumptions.

4 CONCLUTIONS

The shoreline of Lake Hamrin is characterized by numerous curvatures, indentations, and irregularities, as clearly observed in the delineated boundary derived from Sentinel-2 L2A imagery. Under such complex geometrical conditions, the Shoelace Formula provides a more accurate estimation of the water surface area compared to Simpson's Rule. This is because the Shoelace method directly utilizes the Cartesian coordinates of all marked vertices along the shoreline, thereby capturing even small-scale variations and irregular shapes. In contrast, Simpson's Rule is based on numerical integration and tends to approximate the boundary as smooth segments, which may underestimate or overestimate the area when applied to irregular coastlines. Consequently, given the dense set of points digitized along the shoreline, the Shoelace method yields higher accuracy and reliability for Lake Hamrin's water surface area estimation. The shoelace method is more reliable when dealing with lakes with complex and winding shorelines. The Simpson method may be sufficient when the water surface is low or in regular shapes. Finally, Lake Hamrin has been affected, like the rest of the lakes in Iraq, by a decrease in its water level as a result of climate change and the lack of water releases into Iraq's rivers by neighboring countries.

5 RECOMMENDATIONS

Cooperation with neighboring countries to increase Iraq's water share. Relying on satellite imagery (Sentinel, Landsat) to regularly monitor changes in the lake's area. Preventing agricultural and urban encroachments on the lake basin.

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