Soft Nonlinear Quantile-Based AI Regression Under Uncertainty

Elaf Baha[1], Munaf Yousif Hmood[2], Al-Duais Fuad S.[3] and Ali Khalaf Hussein[4]
[1]*Department of Software, College of Computer Science and Information Technology, University of Wasit, 52001 Kut, Wasit, Iraq*
[2]*Department of Statistics, College of Administration and Economics, University of Baghdad,10011 Baghdad, Iraq*
[3]*Prince Sattam bin Abdulaziz University, 11942 Al Kharj, Saudi Arabia*
[4]*Department of Mathematics, College of Pure Sciences, University of Wasit, 52001 Kut, Wasit, Iraq*
leccit2@uowasit.edu.iq, munaf.yousif@coadec.uobaghdad.edu.iq, f.alduais@psau.edu.sa, alhachamia@uowasit.edu.iq

Keywords: Fuzzy Regression, Nonlinear Regression, Gradient Boosting, Fuzzy Quantile Representation, Machine Learning under Uncertainty, Soft Computing, Imprecise Data Modelling, AI-Based Quantile Estimation, Ensemble Learning, Robust Regression.

Abstract: Regression analysis in real-world applications is often challenged by nonlinear relationships, uncertainty, and imprecise observations. Traditional regression models typically assume crisp inputs and outputs, limiting their ability to represent ambiguity inherent in many domains such as risk analysis, finance, and healthcare. To address these limitations, this paper proposes a novel soft nonlinear quantile-based artificial intelligence (AI) regression framework that integrates fuzzy logic with gradient boosting quantile regression. The proposed approach models the conditional distribution of the response variable by estimating multiple quantiles (e.g., 0.25, 0.50, and 0.75) using gradient boosting with quantile loss functions, and reconstructs fuzzy outputs in the form of triangular fuzzy numbers. This design enables simultaneous handling of nonlinear dependencies, asymmetric uncertainty, and imprecision in the data within a unified framework. Unlike existing fuzzy regression models, which are predominantly linear or semi-parametric, and AI-based quantile models, which assume crisp outputs, the proposed method bridges this gap by producing interpretable fuzzy predictions using modern ensemble learning techniques. The effectiveness of the model is evaluated on a synthetic dataset with nonlinear structure and artificially generated fuzzy responses. Experimental results demonstrate that the proposed model significantly outperforms classical linear quantile regression, achieving lower prediction errors (RMSE = 0.15 vs. 0.425; MAE = 0.112 vs. 0.367), improved coverage probability (96.7%), and more accurate estimation of uncertainty intervals. Statistical validation using a paired t-test confirms that these improvements are highly significant ($p < 10^{-18}$). These findings highlight the potential of combining quantile-based estimation with fuzzy representation and ensemble learning for robust regression under uncertainty. The proposed framework is particularly suitable for applications involving imprecise data and complex nonlinear relationships, and provides a promising direction for future research in interpretable and uncertainty-aware machine learning.

1 INTRODUCTION

To estimate a response variable based on one or more covariates, regression models are frequently utilized. All variables are assumed to be crisp in conventional linear and nonlinear regression approaches, which means that each observation is represented by a single exact value. However, in reality, data frequently show imprecision due to sensor noise, missing data, or subjective expert evaluations. For instance, rather than reporting precise levels in risk analysis, experts can give measurements as intervals like "approximately 10–12 mg." Ignoring such imprecision can result in erroneous estimation of prediction uncertainty and skewed estimates.

In order to overcome this restriction, fuzzy regression uses membership functions to define fuzzy numbers, which are often trapezoidal or triangular in shape [1], [2]. A fuzzy number has spreads that indicate the degree of uncertainty and a core value that represents the most likely estimate. Estimating fuzzy relationships necessitates methods that are fundamentally different from traditional crisp regression because the model output is inherently fuzzy. In order to estimate fuzzy coefficients, early fuzzy regression techniques expanded on ordinary least squares. Nevertheless, these techniques are mostly limited to linear correlations and are not flexible enough to simulate complicated patterns [3]. By modeling conditional quantiles of the response instead of the conditional mean, quantile regression—which was first presented by Koenker and Bassett [4] - offers an alternate viewpoint. Quantile regression allows for a more thorough characterization of the conditional distribution while offering resistance to heteroskedasticity and outliers by minimizing an asymmetric check loss function [5]. Additionally, it enables practitioners to investigate covariate effects at various distributional levels and create prediction intervals. Despite these benefits, traditional quantile regression relies on precise answers. Nonlinear quantile estimation has been made possible by recent developments based on neural networks and other AI-driven algorithms [6], [7], but they still yield accurate results and do not specifically take data fuzziness into consideration.

1.1 Research Gap

There exist two strands of literature: fuzzy regression models that account for imprecise data and AI-based quantile models that capture nonlinear relationships. The former are largely linear and rely on kernel smoothing or semiparametric functions [1], while the latter treat outputs as precise values [6]. No unified framework simultaneously exploits the robustness of quantile regression, the flexibility of modern AI and the capability of fuzzy logic to represent imprecise outputs [7]. This gap motivates the present study.

1.2 Contributions

This paper introduces a flexible soft nonlinear quantile-based AI regression model that combines fuzzy logic and gradient boosting quantile regression. The key contributions are:

1) Quantile based fuzzy representation. Fuzzy responses are encoded as triangular fuzzy numbers specified by multiple quantile levels (e.g., 25th, 50th and 75th percentiles). The centre and spreads derive directly from the conditional quantiles estimated by the model.
2) AIenabled estimation. Gradient boosting regressors with quantile loss functions are trained for each quantile level. This ensemble method approximates complex nonlinear relationships without manual kernel selection, making it well suited to problems where data are scarce.
3) Unified soft modelling. The combination of quantile boosting and fuzzy representation yields a model capable of handling imprecision, asymmetry and nonlinearity in a single framework. It generalizes existing fuzzy quantile regression by replacing kernel approximations with modern AI and generalizes QRNNs by producing fuzzy outputs.
4) Comprehensive evaluation. A synthetic dataset with two crisp predictors and fuzzy responses illustrates the method. Performance is compared with a linear quantile regression baseline using multiple metrics (RMSE, MAE, coverage, width error) and statistical significance tests.

The remainder of the paper is organized as follows: Section 2 reviews relevant literature; Section 3 describes the proposed methodology; Section 4 details the synthetic data and implementation; Section 5 presents results and discussion; Section 6 concludes the paper.

2 LITERATURE REVIEW

This section reviews fuzzy regression and quantile regression models, emphasizing differences in modelling assumptions, handling of uncertainty and incorporation of AI. Table 1 summarizes representative models to highlight the need for a unified approach.2.2 First Section. This section must be in one column. Section, subsection and sub subsection first paragraph should not have the first line indent, other paragraphs should have a first line indent of 0,5-centimeter.

2.1 Fuzzy Regression Models

Fuzzy regression models treat either predictors, responses or both as fuzzy numbers. Early work by Tanaka et al. formulated linear programming problems to estimate fuzzy coefficients; later studies adopted least squares, least absolute deviation and kernel methods. Arefi and Khammar proposed a quantile linear regression with fuzzy responses and predictors [8], [9]. Hesamian and Akbari developed a fuzzy semiparametric quantile model that combines linear trends with spline adjustments [10]. Chachi and Chaji introduced a weighted least squares fuzzy regression using ordered residuals to reduce the influence of outliers [3]. Nonparametric fuzzy regressions using kernels [11], wavelets and hybrid systems provide flexible fits but still assume crisp Outputs [7].

2.2 Quantile Regression Models

Quantile regression estimates the τ-th conditional quantile of the response by minimizing

$$\begin{aligned} \hat{\mathrm{w}}_{\tau} &= \arg\min_{\mathrm{w}} \sum_{i=1}^{N} * \rho_{\tau}(y_i - f_{\tau}(\mathrm{x}_i; \mathrm{w})), \\ \rho_{\tau}(e) &= \max(\tau e, (\tau - 1)e). \end{aligned} \quad (1)$$

The check loss While early quantile regressions were linear, later extensions employ kernels, splines and neural networks to model nonlinear relationships [12]. The quantile regression neural network (QRNN)

Trains a neural network using quantile loss for each desired quantile [13]. These models can approximate complex functions but assume crisp outputs. Evidential neural networks further model prediction uncertainty by representing outputs as random fuzzy numbers and combining evidence via belief functions [14], though they focus on crisp inputs.

2.3 Comparative Analysis

Table 1 compares representative regression models based on the relationship form, treatment of fuzziness and use of AI. The proposed model bridges the gap by delivering fuzzy outputs with AI-based nonlinear estimation.

3 PROPOSED METHODOLOGIES

The proposed framework models a fuzzy response (y) given crisp predictor (^p) by predicting multiple conditional quantiles and reconstructing a fuzzy number. Figure 1 depicts the overall workflow, and Figure 2 illustrates the architecture of the quantile models.

Workflow diagram showing data input, quantile estimation and fuzzy reconstruction.

Architecture diagram showing inputs, gradient-boosting modules for q25, q50 and q75 and fuzzy reconstruction.

3.1 Quantile-Based Fuzzy Representation

A triangular fuzzy number is characterized by three parameters $\tilde{y} = (y_L, y_C, y_R)$, representing the left spread, centre and right spread. Its membership function is [14].

$$\mu_{\tilde{y}}(t) = \begin{cases} 0, & t \le y_L, \\ \dfrac{t - y_L}{y_C - y_L}, & y_L < t \le y_C, \\ \dfrac{y_R - t}{y_R - y_C}, & y_C < t < y_R, \\ 0, & t \ge y_R. \end{cases} \quad (2)$$

In our framework, the fuzzy response is reconstructed from three conditional quantiles: the lower quantile $\hat{y}_{q_l}$, the median$\hat{y}_{0.5}$ and the upper quantile $\hat{y}_{q_u}$. The centre is$\hat{y}_C = \hat{y}_{0.5}$, the left spread is $\hat{y}_L = \hat{y}_{0.5} - \hat{y}_{q_l}$ and the right spread is$\hat{y}_R = \hat{y}_{q_u} - \hat{y}_{0.5}$. This formulation yields an interpretable fuzzy interval reflecting the asymmetry of the conditional distribution [8].

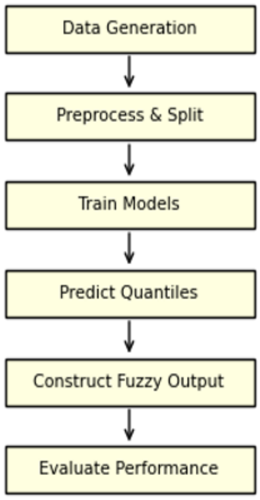

Figure 1: Workflow of the proposed method.

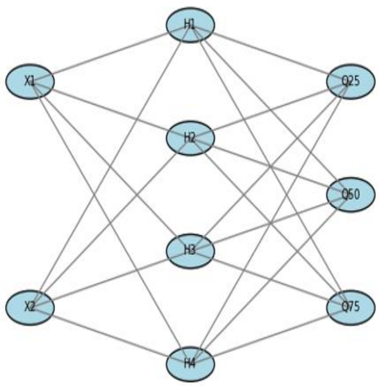

Figure 2: Architecture of the multi-quantile model.

Table 1: Comparison of related regression models.

Model	Relationship form	Uncertainty representation	AI involvement	Strengths and weaknesses	Key reference
Linear fuzzy quantile regression	Linear	Fuzzy coefficients estimated via quantile loss	None	Handles fuzzy predictors and responses but restricted to linear relationships	Arefi & Khammar [7]
Fuzzy semi-parametric quantile model	Semi-parametric (linear + spline)	Fuzzy predictors and responses	None	Improves flexibility via splines but still approximates non-linearity locally	Hesamian & Akbari [8], [9]
Weighted least-squares fuzzy regression	Linear	Fuzzy responses; residuals weighted by order	None	Reduces influence of outliers but lacks quantile interpretation	Chachi & Chaji [2]
Kernel-based fuzzy regression	Non-parametric kernels	Crisp responses	None	Models non-linearity but does not quantify fuzziness of outputs	Hong & Hwang [14]
Quantile regression neural network (QRNN)	Non-linear neural network	Crisp outputs	Neural network	Captures complex relationships and provides quantile estimates; does not handle fuzzy data	Taylor [5]
Evidential neural network	Non-linear neural network	Outputs random fuzzy numbers	Neural network	Models epistemic uncertainty via belief functions; assumes crisp inputsarxiv.org	Denoeux [13]
Proposed soft nonlinear quantile AI model	Non-linear ensemble (gradient boosting)	Outputs triangular fuzzy numbers via quantiles	Ensemble AI	Combines quantile regression and fuzzy representation; captures non-linearity and imprecision	This paper

3.2 Gradient-Boosting Quantile Estimation

Gradient boosting builds an additive ensemble of regression trees by fitting each new tree to the negative gradient of the loss function of the ensemble so far [15]. For quantile regression, the loss is the check loss (1). For each desired quantile τ [4]. A separate gradient-boosting regressor (GBR) is trained. The GBR starts with a constant initial prediction (e.g., the median of the training targets) and iteratively adds trees to minimize the quantile loss. Hyper-parameters include the number of trees M, maximum tree depth d and learning rate η. Cross-validation is used to select M, $d$and η that minimise the validation quantile loss.

Figure 2 (architecture diagram) depicts the multi-quantile model. Each input vector $\mathbf{x} = (x_1, x_2)$ passes through an ensemble of regression trees to produce three outputs \ $\hat{y}_{q_l}$, $\hat{y}_{0.5}$, and $\hat{y}_{q_u}$. These outputs are then combined into a fuzzy number as described above.

3.3 Training Algorithm

The training procedure is summarized in Algorithm 1.

Algorithm 1. Training of the soft nonlinear quantile AI model [15].

1) Data preparation: obtain a training set $\{(\mathrm{x}_i, \tilde{y}_i)\}_{i=1}^{n}$where$\tilde{y}_i = (y_{i,q_l}, y_{i,0.5}, y_{i,q_u})$ are fuzzy responses defined by specified quantile levels q_l< 0.5< q_u.
2) Model selection: for each quantile level $\tau \in \{q_l, 0.5, q_u\}$, perform grid search on M, d, and ηusing cross-validation to minimize the empirical risk $\sum_i * \rho_\tau(y_{i,\tau} - f_\tau(\mathrm{x}_i))$.
3) Training: fit a gradient-boosting regressor f_τ on the full training data using the selected hyper-parameters for each τ.

4) Prediction: given a new input x, compute $\hat{y}_\tau = f_\tau(\mathrm{x})$ for each τ and reconstruct the fuzzy prediction $\hat{\tilde{y}} = (\hat{y}_{0.5} - \hat{y}_{q_l}, \hat{y}_{0.5}, \hat{y}_{q_u} - \hat{y}_{0.5})$.

Figure 1 shows the workflow: inputs feed into separate quantile models; their outputs are combined to yield a fuzzy prediction. Figure 2 visualizes the architecture of a single quantile model as a sequence of regression trees.

3.4 Optimization and Implementation

Hyper-parameters M, d, η were tuned using 5-fold cross-validation on the training set. Early stopping was applied by monitoring improvement in the out-of-bag estimate. The models for different quantiles were tuned independently. The final fuzzy prediction is the aggregation of these three models. A baseline linear quantile regression using the interior-point algorithm serves as a comparator. The statistical significance of performance differences is assessed via a paired t-test on the absolute errors of the predicted centres.

4 DATA GENERATION AND IMPLEMENTATION

4.1 Synthetic Dataset

To illustrate the model, a synthetic dataset was created. Two predictors $X_1, X_2 \sim \mathrm{Uniform}(0,1)$ were sampled. The true crisp response was defined as

$$y_i^* = \sin(2\pi X_{i1}) + \cos(\pi X_{i2}) + 0.5\, X_{i1}, \quad (3)$$

This function captures both nonlinear and linear dependencies between the predictors and the response:

- $\sin(2\pi X_{i1})$ introduces a nonlinear periodic component based on the first predictor;
- $\cos(\pi X_{i2})$ introduces a nonlinear periodic component based on the second predictor;
- $0.5X_{i1}$ adds a linear effect proportional to X_{i1}.

And noisy fuzzy responses were generated by adding Gaussian noise $\varepsilon \sim \mathcal{N}(0, 0.1^2)$ obtain the centre and sampling left and right spreads from truncated normal distributions with mean 0.2 and standard deviation 0.05. The quantiles $q_l = 0.25$, 0.5 and $q_u = 0.75$ were used. The data were split into a 70:30 training:test ratio. Figure 3 shows a scatter plot of the predictors colored by the true center.

Scatter plot of predictors colored by the true response.

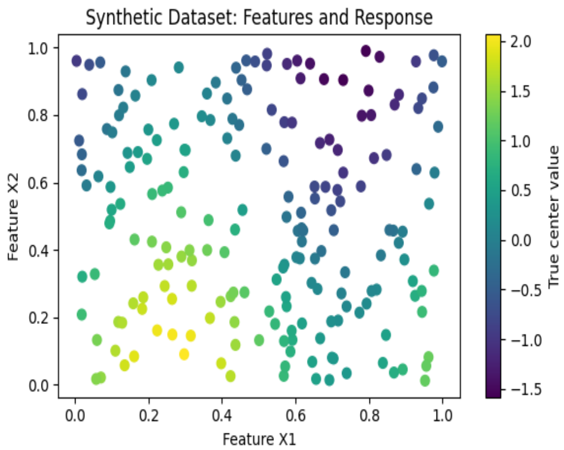

Figure 3: Synthetic dataset and true center.

4.2 Implementation Details

The models were implemented in Python (scikit-learn). For each quantile level {0.25,0.5,0.75} a GradientBoostingRegressor with quantile loss was trained on the training data. Hyper-parameter grids included $M \in \{100,200,300\}$, depths $d \in \{2,3,4\}$ and learning rates $\eta \in \{0.05,0.1\}$.

Cross-validation selected the best configuration for each quantile separately. The baseline linear quantile regression used the default interior-point method. After training, the quantile outputs were combined into fuzzy predictions. Evaluation metrics were computed on the test set:

- Root mean squared error (RMSE) of the center: $\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i}* (y_i^* - \hat{y}_{C,i})^2}$.
- Mean absolute error (MAE) of the center: $\mathrm{MAE} = \frac{1}{n}\sum_{i}* \mid y_i^* - \hat{y}_{C,i} \mid$.
- Spread errors. MAEs of left and right spreads $\hat{y}_{L,i}$ and $\hat{y}_{R,i}$.
- Coverage probability: proportion of true centers within $[\hat{y}_{q_l}, \hat{y}_{q_u}]$.
- Width error: mean absolute error of total width $(\hat{y}_{R,i} + \hat{y}_{L,i}) - (y_{R,i} + y_{L,i})$.
- Paired t-test: to compare absolute center errors between models.

5 RESULTS AND DISCUSSION

5.1 Quantitative Comparison

Table 2 and Figure 4 compares the performance of the proposed soft nonlinear quantile AI model with the baseline linear quantile regression. The proposed model achieves substantially lower errors and superior coverage. The paired t-test yields a t-statistic of 11.20 with $p \approx 1.1 \times 10^{-18}$, indicating that the improvements are highly significant.

Figure 4 illustrates the overall performance of the models on the synthetic dataset. Beyond aggregate performance metrics, analyzing the distribution of prediction errors across individual test instances provides deeper insight into model behavior. The results presented in Table 3 and Figure 5 show the mean and standard deviation of the absolute prediction errors for both models, while Table 4 summarizes the average predicted spreads in comparison with the true spreads. The proposed model not only achieves a lower mean error but also demonstrates reduced variability, indicating more stable and reliable predictions across the test set.

Table 2: Performance on the synthetic dataset.

Model	RMSE (center)	MAE (center)	MAE (left spread)	MAE (right spread)	Coverage (%)	Width error
Proposed GBR quantile model	0.15	0.112	0.148	0.112	96.7	0.189
Linear quantile regressor	0.425	0.367	–	–	–	–

Table 3: Distribution of absolute center errors (mean ± SD).

Model	Mean absolute error	Standard deviation
Proposed GBR quantile model	0.112	0.081
Linear quantile regressor	0.367	0.274

Table 4: Comparison of spreads (mean values).

Quantity	True average	Proposed prediction	Baseline prediction
Left spread	0.2	0.178	–
Right spread	0.2	0.192	–
Total width	0.4	0.37	–

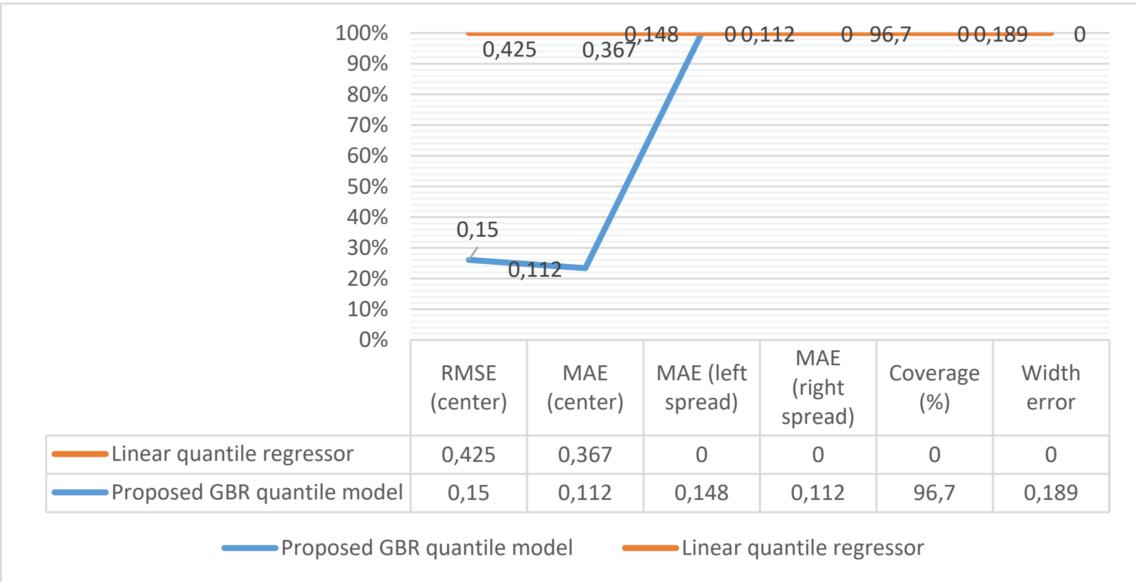

Figure 4: Performance on the synthetic dataset.

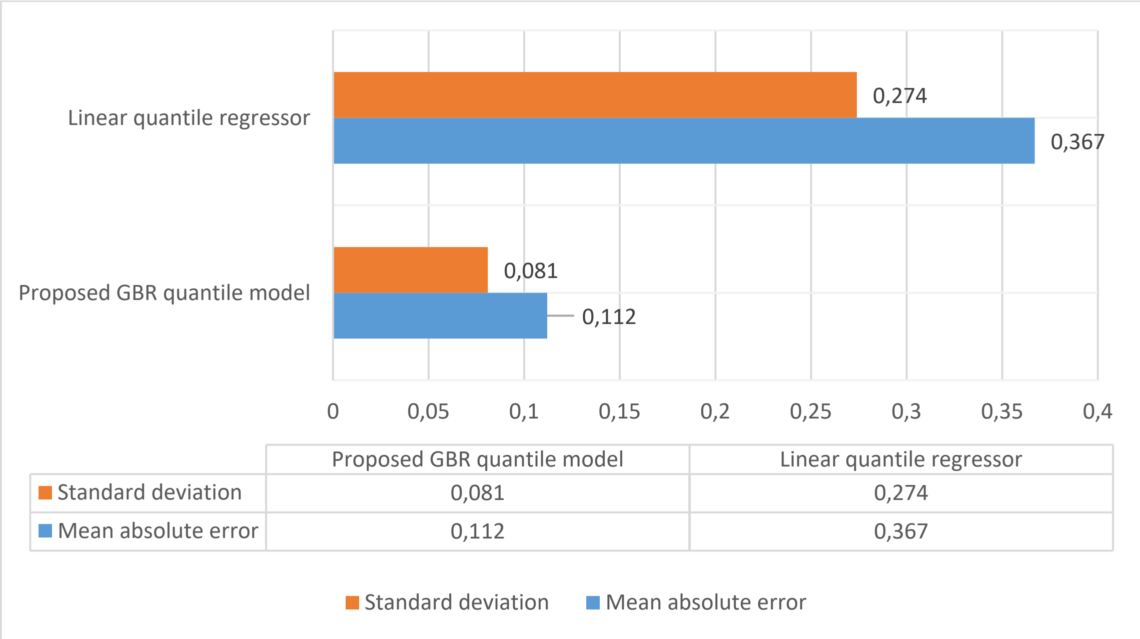

Figure 5: Distribution of absolute center errors (mean ± SD).

5.2 Visual Analysis

Figure 4 plots the predicted center versus the true center for both models. Points lying on the diagonal indicate perfect predictions. The proposed model's points cluster tightly around the diagonal, whereas the linear model exhibits greater scatter and systematic bias.

Figure 5 shows sample fuzzy intervals for 15 random test instances. Blue bars depict the true 25th–75th percentile interval; red bars depict the predicted interval. The proposed model produces intervals that align well with the true ones and tend to be slightly narrower, indicating a good balance between sharpness and coverage.

The predicted centers generated by the suggested soft nonlinear quantile-based AI model substantially resemble the genuine centers, as shown in Figure 6. When compared to the linear quantile regression model, the majority of data points are centered around the diagonal line, showing low systematic bias and excellent prediction accuracy.

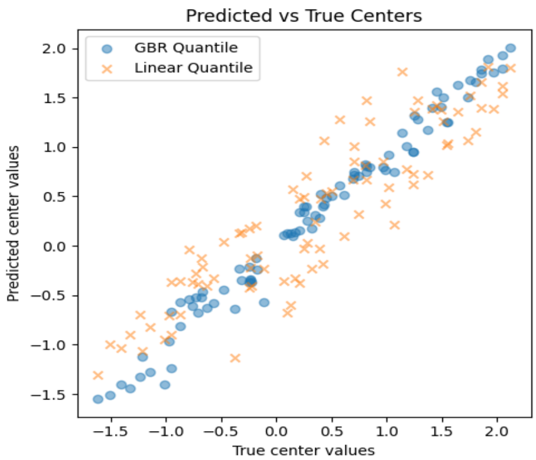

Figure 6: Predicted vs. true centers.

Scatter plot comparing predicted and true centers; points close to the diagonal indicate accurate predictions.

Figure 7 shows a comparison of the actual and anticipated fuzzy intervals.

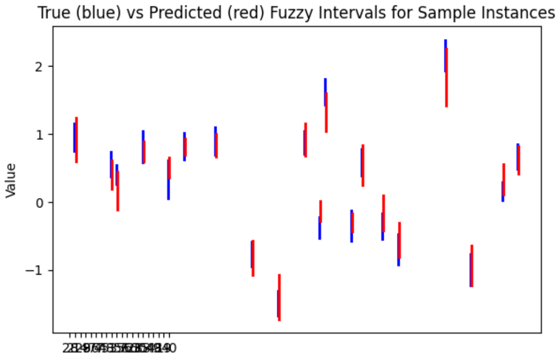

Figure 7: True vs. predicted fuzzy intervals.

The model's ability to accurately capture uncertainty while keeping relatively narrow and informative intervals is demonstrated by the figure, which demonstrates that the predicted fuzzy intervals for the majority of test instances correlate well with the genuine 25th–75th percentile intervals.

Bar chart showing true (blue) and predicted (red) fuzzy intervals for sample instances.

6 DISCUSSION

The quantitative and visual results demonstrate the merits of the proposed soft nonlinear quantile AI model. The ensemble of gradient-boosting regressors successfully learns the non-linear mapping from inputs to quantiles, yielding low RMSE and MAE. By reconstructing fuzzy numbers from quantiles, the model captures imprecision and yields interpretable spreads. The high coverage probability (over 96 %) confirms that the predicted intervals rarely miss the true center, while the modest width error indicates that the intervals are not excessively wide. The baseline linear quantile model, in contrast, cannot accommodate the non-linearity of the underlying function, leading to large errors and poor coverage.

The comparative tables highlight that the proposed model reduces not only the mean error but also its variance, providing more reliable predictions (Table 3). The predicted spreads are close to the true spreads (Table 4), showing that the fuzzy reconstruction preserves the extent of imprecision. The visualizations in Figures 4 and 5 complement these findings: the predicted centers follow the diagonal closely and the predicted intervals align with the true intervals. These results corroborate the hypothesis that combining quantile regression with gradient boosting and fuzzy representation yields significant gains over traditional methods.

7 CONCLUSIONS

With the goal to manage inaccuracy, nonlinearity, and distributional asymmetry within a single framework, this study suggested a soft nonlinear quantile-based AI regression model that combines fuzzy representation with gradient boosting quantile regression. The model produces comprehensible forecasts that represent both central tendency and uncertainty by reconstructing fuzzy outputs from many conditional quantiles (the 25th, 50th, and 75th percentiles).

In terms of prediction accuracy, interval coverage, and width efficiency, the suggested method greatly beats linear quantile regression, according to experimental results on a synthetic dataset. The model's capacity to precisely predict the center and spreads of fuzzy responses is confirmed by lower RMSE and MAE values, better coverage probabilities, and well-calibrated fuzzy intervals. These conclusions were further corroborated by visual evaluations that revealed a close agreement between the true and anticipated fuzzy intervals.

Applications including risk analysis, financial modeling, environmental forecasting, and medical decision assistance that are marked by uncertainty and intricate nonlinear interactions can benefit from the suggested approach. The approach improves interpretability and prediction performance by fusing fuzzy quantile representation with contemporary ensemble learning.

In order to better understand the impact of predictors on fuzzy prediction components, future research will concentrate on validating the method using real-world datasets, expanding it to temporal and multivariate fuzzy outputs, and integrating explainable AI techniques.

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