Mathematical Modeling of Mixers with Cavitation Inducements

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Abstract: The object of research in this article is the hydrodynamic processes in continuous cavitation devices and the design parameters of cavitators. The aim of the research is to develop an optimal hydrodynamic cavitation mixer for oil activation and production of water-fuel emulsions from petroleum products. The optimal design parameters of cavitation mixing devices with a hydrodynamic grid have been determined. The calculation of the mixer with minimal energy consumption and optimal workflow is carried out by the method of successive approximations. A method for calculating hydrodynamic cavitation mixers is proposed, based on the proposed mathematical model of the workflow, together with the conditions for minimal pressure loss in the device and recommendations for its optimal design, which allows the development of efficient continuous mixing devices with minimal energy consumption. It is recommended that, according to the proposed method, resource-saving industrial samples of cavitation mixers of a new generation for various volume costs of a water-fuel mixture will be calculated in the future.

1 INTRODUCTION

Currently, cavitation technologies give excellent results in the transformation of gaseous, solid and liquid media. These technologies are used to prepare mixtures resistant to segregation (also for mixing difficult-to-mix or immiscible media), homogeneous solutions, emulsions, suspensions and dispersions from various products, to activate enzymes and accelerate processes by supporting catalytic reactions, to treat wastewater, and also for water purification in water treatment systems [1]-[2].

The working process of a cavitation mixer is based on the phenomena that occur during the joint flow of liquid and vapor-gas phases, and, in the general case, the speeds of movement of liquid and vapor (gas) are different [3]-[5]. As a result, the data on the flow rate of the medium, the geometry of the channel, and the physical properties of the liquid and gas (steam) do not yet provide a sufficiently complete idea of the hydrodynamics of the flow. Therefore, to characterize a two-phase flow, it is necessary to introduce quantities that take into account the features of the movement of individual phases.

The mathematical model of the working process, compiled on the basis of the material, energy and heat balance of the flow in continuous mixers, is closed [4]. It makes it possible to calculate the transverse dimensions of mixers with cavitation inducers in the flow in the form of a multi-jet nozzle or a hydrodynamic cavitation grate, as well as to determine the coordinates of the mixing shock and, thus, the longitudinal dimensions of the mixer.

Related problems related to cavitation technologies and flow hydrodynamics were considered in [6]-[15].

2 METHOD AND RESULTS

In this work, the gas-liquid flow is considered as pseudo-one-dimensional, its parameters are averaged over the pipe section. All effects from the uneven distribution of parameters over the pipeline cross section are taken into account in the dependencies for averaged parameters, temperature effects are not taken into account. It is assumed that the viscosity of the phases manifests itself only in the processes of interfacial interaction and does not manifest itself in the macroscopic momentum transfer, the pressure in the phases is the same, the flow occurs in pipes with a constant area.

Bubble, plug, and emulsion structures are combined into a single flow structure, and conventional single-velocity models were used for their calculation [3]. Usually, these regimes exist at gas content.

When modeling a dispersed-annular flow, it was assumed that the velocities of the gas and droplets are equal to each other and differ from the film velocity.

The calculation of the parameters of this mixture flow regime was carried out using a system of differential equations [3] consisting of three continuity equations (for gas, droplets, and liquid) and two momentum equations (for a film and a gas-droplet core of the flow), which take into account the intensity of droplet deposition on the film, intensity of dynamic entrainment of droplets from the film surface:

$$\begin{aligned}
&\frac{\partial}{\partial t}(\rho_1^o\alpha_1)+\frac{\partial}{\partial z}(\rho_1^o\alpha_1 v_1)=J_{21}+J_{31},\\
&\frac{\partial}{\partial t}(\rho_2^o\alpha_2)+\frac{\partial}{\partial z}(\rho_2^o\alpha_2 v_1)=-J_{21}-J_{23}+J_{32},\\
&\frac{\partial}{\partial t}(\rho_2^o\alpha_3)+\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3)=-J_{31}+J_{23}-J_{32},\\
&\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3)+\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3^2)=-\alpha_3\frac{dP}{dz}-\\
&-F_w+F_{13}-\rho_2^o\alpha_3 g-(J_{32}+J_{31})v_3+J_{23}v_1\\
&\frac{\partial}{\partial t}\left[(\rho_1^o\alpha_1+\rho_2^o\alpha_2)v_1\right]+\frac{\partial}{\partial z}\left[(\rho_1^o\alpha_1+\rho_2^o\alpha_2)v_1^2\right]=\\
&-(\alpha_1+\alpha_2)\frac{\partial P}{\partial z}-F_{13}-\\
&-(\rho_1^o\alpha_1+\rho_2^o\alpha_2)g-J_{23}v_1+(J_{31}+J_{32})v_3.
\end{aligned} \qquad (1)$$

Here, the parameters related to the gas phase, drops and liquid film contain subscripts 1, 2, 3, respectively, t-time, z-coordinate, P-pressure, α_i - volume concentration of the i-th component of the mixture, v_i - speed of the i-th component mixture, ρ_i – the density of the i-th component of the mixture, g – the acceleration of free fall, J_{21}, J_{31} - the intensity of the inflow of gas and liquid through the permeable walls of the pipe, J_{32}, J_{23} - the intensity of the entrainment of drops from the surface of the film and the settling of drops, F_{13}, F_w - the force interaction between the film and drops, as well as the wall of the emulsifier .

The initial equations describing the working process in the mixer are [16]-[17]:

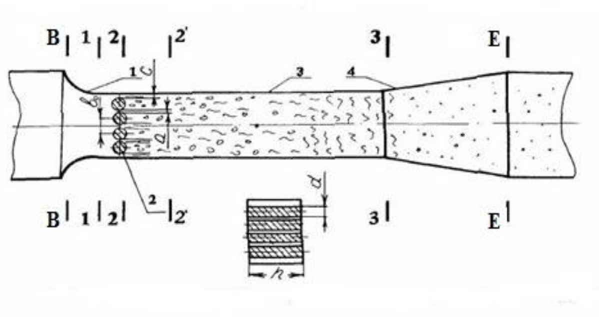
1) cost balance equation;
2) D. Bernoulli's equation for a flow with a droplet state of the mixture;
3) the balance equation for the specific energy of the flow with the droplet state of the mixture;
4) momentum equation for the control compartment.

The combination of these equations and a number of transformations, taking into account the equality of the areas of the normal sections of the flow $A_B=A_E$ and $A_1=A_3$, lead to the expression

$$\frac{P_B-P_E}{P_B-P_{S.S.}}=\frac{\zeta_{NOZZ}+(\zeta_{CONF}+\zeta_{DIFF}+\zeta_{\Gamma})\Omega^2+(1-\Omega)^2}{1+\sigma+\zeta_{NOZZ}+\zeta_{CONF}\cdot\Omega^2} \qquad (2)$$

which is basic when calculating a mixer with a multi-jet nozzle (Fig. 1).

Expression (2) allows, with a known absolute pressure in front of the mixer PB and the selected resistance coefficients of the elements of the flow path, to determine the relative area of the nozzle Ω, at which the pressure loss in the mixer (PB– PE) will not exceed the specified value.



1- Confuser; 2- lattice ; 3- working chamber (neck); 4- diffuser

Figure 1: Schematic diagram of a mixer wich a hudrodynamic grate.

On Figure 2 shows graphs of the change in the relative pressure drop (PB–PE) / (PB– PS.S.) as a function of the relative area of the nozzle Ω, calculated according to (2), in the entire practical range of change in the resistance coefficient of the working chamber (throat) = 0.08...1 and hydraulically perfect profiling of the remaining elements of the flow part of the mixer ξCONF = 0.15; ξNOZZ = 0.10; ξDIFF = 0.25 [18].

An analysis of expression (2) and graphs (see Fig. 2) showed that the minimum relative pressure drop (PB–PE) / (PB–PS.S.), and therefore, the minimum loss in the mixer corresponds to the following range of change in the relative area of the nozzle 0.45 <Ω<0.7. Therefore, when calculating hydrodynamic cavitation mixers, it is advisable to choose the geometric characteristics from this series.

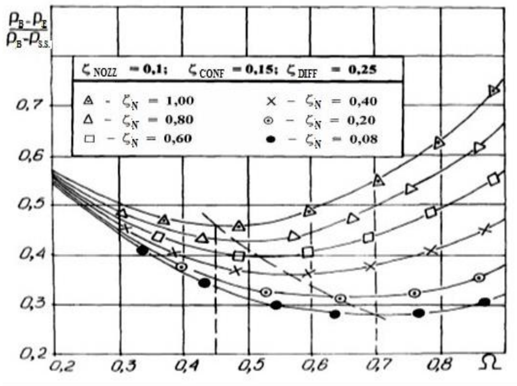


Figure 2: Graph of change in relative pressure drop depending on the relative area of the nozzle.

The task of calculating the mixer is as follows: with known physical properties of the mixture components (carrier medium and liquid additive), their content in the mixture flow, initial pressure and temperature, the operating and geometrical parameters of the mixer are determined, realizing the flow of the working process with minimal energy consumption. The calculation of the mixer with minimum power consumption and optimal working process is performed by the method of successive approximations in the Matlab application software package. At the same time, in each approximation, according to the initial data and the selected hydraulic resistance coefficients of the cavitators (nozzle or hydrodynamic grate) and elements of the flow part of the working chamber, the optimal transverse dimensions of the cavitation stimulators and the working chamber are determined, corresponding to the minimum losses in the mixer, then the steam flow rate and then the flow temperature are calculated mixture in the initial section of the working chamber T_2. The calculation is considered reliable if the flow parameters of the mixture of components in the last two approximations differ by no more than (2...5) %.

In this case, the total axial lengththe working chamber consists of sections:

- formation of a supersonic vapor-gas-liquid flow lform;
- rapid supersonic flow of the mixture lflow;
- mixing jump ljump, where there is an abrupt transition from supersonic to subsonic flow and condensation of liquid additive vapors;
- calming the flow lcalm.

$$L_N = l_{form} + l_{flow} + l_{jump} + l_{calm} \quad (3)$$

According to the recommendations [5], the formation areasupersonic vapor-liquid flow lform= (1 ... 2) D_3, where D_3 -working chamber diameter. Extent of the supersonic flow zonelflowis determined by the critical length l_{kr} of the section with a turbulent two-phase flow, at which the critical state of the flow is reached in the end section ($P = P_{KR}$) and is determined according to [16], i.e. $l_{flow} = l_{kr}$. The length of the mixing jump is l_{jump}= (2 ... 4) D_3. The calming zone can be recommended in the range lcalm = (2...3) D_3.

Based on the obtained mathematical model [3] and the methodcalculation, let's calculate the mixer with a multi-jet nozzle, which serves to create a water-oil emulsion (WME) (Fig. 1).

Determine the density of the mixture:

$$\rho_m V_m = \rho_{ad} V_{ad} + \rho_{ca} V_{ca}$$

Here:

- V_m=1 is the volume of WMS;
- V_{ca}=0.85 – volume of fuel oil in WMS;
- V_{ad}=0.15 is the volume of water in the WMS.

$\rho_m = 0.15\rho_{ad} + 0.85\rho_{ca} = 0.15 * 958.4 + 0.85 * 949 \cong 950$kg/m^3.

We will calculate the emulsifier by the method of successive approximations in the Matlab application software package. As a first approximation, we take the following values of hydraulic resistance coefficients:

$$\zeta_{CONF} = 0.15; \ \zeta_{NOZZ} = 0.10;$$
$$\zeta_{DIFF} = 0.25; \ \zeta_N = 0.2;$$

The minimum relative pressure drop at the selected hydraulic resistance coefficients, and, consequently, the minimum loss in the mixer, corresponds to Ω = 0.64. With a relative nozzle area Ω=0.64, the cavitation number will be equal to σ = 0.41. The function minimum corresponds to the minimum pressure loss in the mixer $\Delta P_{BE} = P_B - P_E$.

At absolute pressure in the initial section P_B = 1000 kPa and saturated vapor pressure P_B = 101 kPa, the minimum pressure loss $\Delta P_{BE} = P_B - P_E = 272 кПа$.

Now determine the speed of the VMS outflow from the nozzle V_{m2}:

$$V_{m2} = \sqrt{\frac{2(P_B - P_{S.S.})}{\rho_m(1 + \sigma + \zeta_{CAV} + \zeta_{CONF}\Omega^2)}}$$

Substituting into the last expression the accepted values of the coefficientsresistance, cavitation number σ, as well as Ω = 0.64, we obtain that in the first approximation, the outflow velocity V_{m2} = 34.71 m/s. Then the speedmixture flow in sections 1-1 and 3-3

$$V_{m1} = V_{m3} = \Omega V_{m2} = 0.64 * 34.71 = 22.21 \text{ m/s.}$$

Let's check the pressure loss in the mixer:

$$\Delta P_{BE} = \zeta_{CAV} \frac{\rho_m V_{m2}^2}{2} + (\zeta_{CONF} + \zeta_{DIFF} + \zeta_N) \frac{\rho_m V_{m3}^2}{2} + \frac{(V_{m2} - V_{m3})^2}{2} \rho_m =$$

$$0.1 \frac{950 * 34.71^2}{2} + (0.15 + 0.25 + 0.2) \frac{950 * 22.21^2}{2} + \frac{(34.71 - 22.21)^2}{2} 950 = 2{,}7206 * 10^5 Pa$$

3 DISCUSSION

Comparing the obtained result with the one calculated earlier [17], we find that these results are almost the same. This indicates that the velocities V_{m2} and V_{m3} are found correctly. Knowing the speed V_{m3} of the emulsion flow in the neck, we determine its area A_3 and diameter D_3 in the first approximation. Jet area

$$A_2 = \Omega * A_3 = 0.64 * 1.50 * 10^{-3} = 0.96 * 10^{-3} m^2$$

$$A_3 = \frac{Q_m}{V_{m2}} = \frac{3.33 * 10^{-2}}{22.21} = 1.50 * 10^{-3} \text{ m}^2$$

A diameter:

$$D_3 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 * 1.50 * 10^{-2}}{\pi}} = 4.37 * 10^{-2} m \cong 44 mm$$

Let us determine the amount of released steam, taking the value slip coefficient Ψ = 0.9 and assuming that steam occupies all jet-free space in the initial section of the working chamber.

Steam movement speed at Ψ = 0.9

$$V_S = \Psi * V_{m2} = 0.9 * 34.71 = 31.24 m/\text{s}$$

Then the amount of steam

$$m_{S_2} = \rho_S V_S (\text{A}_3 - \text{A}_2) = 0.598 * 31.24 (1.50 * 10^{-3} - 0.96 * 10^{-3}) \cong 10^{-2} kg/\text{s}$$

Taking into account expression (3) and the accepted recommendations, we determine the length of the working chamber of the mixer:

$$L_N = l_{form} + l_{flow} + l_{jump} + l_{calm} = 2D_3 + l_{kr} + 4D_3 + 3D_3 = 2 * 0.044 + 0.074 + 4 * 0.044 + 3 * 0.044 \cong 0.47 m$$

Table 1: The main geometric dimensions of the mixer.

Area, m 2		Diameter, m			Length, m	
A2	A3	D2	D3	D4	Lг	Lдиф
9,6·10-4	1,5· 10-3	0,0 156	0,04 4	0,108	0,4 7	0,52 3

$$D_4 = D_3 \sqrt{n} \tag{4}$$

$$L_{DIFF} = 0.5 \frac{(\text{D}_4 - \text{D}_3)}{tg \alpha/2} \tag{5}$$

Other dimensions of the mixer are defined and presented in the Table 1. The number of jets is assumed to be m = 5. To achieve the minimum energy loss in the diffuser, it is assumed that the diffuser opening angle is α= 7 °, and the degree of its expansion is n = 6. Then the diameter of the outlet section of the diffuser is determined by the expression (1) and the axial length of the diffuser is determined by (2).

4 CONCLUSIONS

The mathematical model of the working process, compiled on the basis of the material, energy and heat balance of the flow in continuous mixers, is closed. It allows calculating the transverse dimensions of mixers with cavitation inducers in the flow in the form of a multi-jet nozzle or a hydrodynamic cavitation grate, as well as determining the coordinates of the mixing jump and, thus, the longitudinal dimensions of the mixer [19]-[21]. According to the proposed method, resource-saving industrial samples of cavitation mixers for various volumetric flow rates of the new generation of naval forces will be calculated in the future.

Calculation method for hydrodynamic cavitationmixers, built on the basis of the proposed mathematical model of the workingprocess together with the conditions for minimum pressure loss in the deviceand recommendations for its optimal design, allows the developmentbuild efficient continuous mixing devices withminimal power consumption.

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