

Computer-Oriented Mathematical Modeling of Multiparameter Converters Using Quadratic, Cubic, and N-Dimensional Interpolation

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Abstract: The current stage of scientific and technological progress in all areas of industrial production and management is characterized by the introduction of new information technologies that determine the increased integration of all automated equipment types, from sensors and primary measuring transducers (MT) of a wide range of physical quantities to multi-level production control systems and even entire industries. The presence of an adequate MT model that optimally combines such properties as accuracy, universality of structure, and availability for effective computer implementation is the basis for solving problems of synthesizing MT designs and circuits with specified properties, building economical and productive computing devices as components of control, monitoring, identification, and diagnostic systems. The article considers the problem of creating methods and tools for model support of the design processes of multi-parameter MT of devices for monitoring physical quantities, including the creation of adequate mathematical models of processes in MT, oriented toward computer implementation. Methods of quadratic, cubic, and n-dimensional interpolation of multi-parameter transducers based on the theory of simplexes and barycentric coordinates are proposed. Using theorems, the interpolating quadratic form of n variables in the local barycentric coordinate system is determined, the error of the quadratic interpolation method is estimated, and the type of cubic interpolation polynomial is determined. By selecting the approximation method, optimization of the mathematical description is achieved, which allows reducing the cost of experiments, using effective algorithms for processing measurement information, computer implementation. The practical value of the research in this article is determined by the wide possibilities of technological use of the developed methods, algorithms in computer systems for designing MT, the prospects for the implementation and application of the proposed models and devices in highly efficient multi-level distributed control and management systems, as components of complex integrated control systems for technological processes and production.

1 INTRODUCTION

The development of modern automated and automatic systems of measurement, observation, control, diagnostics and management is characterized by a significant expansion of the scope of applications, the complication of the tasks to be solved, and therefore the processes of functioning of the systems, as well as increased requirements for the accuracy and speed of methods and means of digital signal processing. At the same time, the improvement of methods and means of measurement, the emergence of new principles of control and diagnostics of technical objects using certain physical

effects leads to the need to create and study increasingly effective mathematical methods of signal processing (experimental data), oriented towards computer implementation. Taking into account the growing requirements for methods and means of signal processing, the growth of the volume of calculations can be ensured by creating and improving the corresponding methods of mathematical modeling, as well as computational methods and algorithms implementing mathematical models.

Similar problems of developing mathematical models of signal processing processes, numerical algorithms and programs are typical for modern and

promising research and development in the field of sound location problems, radar systems, seismic information processing, computer tomography, non-destructive testing of technical products, in multi-channel problems of constructing monitoring systems, etc.

Improving and creating new types of measuring transducers requires the use of modern scientific achievements in the field of mathematical modeling, design automation, computational and full-scale experiments. [1] - [4].

Currently, in almost all areas of technology, modeling is a necessary element in the process of creating, testing and introducing new technology objects. With the advent of complex technical systems, the role of modeling in assessing the parameters of the processes under study has increased significantly. This is explained by the peculiarities of the objects under study, resulting from the complexity of functional connections between system parameters, changing environmental conditions and assessed indicators. Usually, when modeling complex systems, we are faced with a situation where the processes under study in the system and environmental conditions are probabilistic in nature, the number of factors influencing the estimated indicators is significant, and estimates of the required parameters need to be obtained for a wide range of changes in the operating conditions of the system. The ultimate goal of mathematical modeling is to achieve the required accuracy of estimates of selected quantitative indicators [5] - [10].

Multiparameter transducers are designed to measure physical and chemical parameters and alarm systems in automatic monitoring and control systems of objects in the electric power industry, technological processes and production and to issue output signals in the form of DC power and voltage, as well as non-electrical quantities converted into DC electrical signals or active resistance. The tasks of converting and ensuring the accuracy of calculations are essential to ensure the required metrological characteristics of multi-parameter measuring and information systems [11] - [13].

The approximation of the transformation function of a multiparameter transducer should provide maximum approximation accuracy with a minimum number of nodes (experimental data); be simple enough so that the technical implementation of the computational processing algorithm is economical; provide the ability to quickly and sufficiently test multi-parameter transducers [14] - [18].

2 MATERIALS AND METHODS

Consider a multiparameter transducer or device, the input of which is supplied with an n-dimensional quantity $\vec{x} \in E^n$, and the output is a one-dimensional quantity u , which is a function of n variable input coordinates $u = u(\vec{x}) \equiv u(x_1, x_2, \dots, x_n)$.

The error in estimating vector quantities significantly depends on the accuracy of approximation of the transformation function of a multiparameter transducer.

If the error of linear interpolation, determined by the relation

$$L(\vec{x}) - u(\vec{x}) = (\vec{p}, \vec{x}) + p_0 - u(\vec{x}) = \frac{1}{2} \sum [H(y^v)(\vec{x} - \vec{x}^v), \vec{x} - \vec{x}^v] \lambda_v(\vec{x}), \quad (1)$$

is large, then it is advisable to move to a higher order of interpolation, for example, use quadratic, cubic and n-dimensional interpolation [19].

The general approach developed in this work based on the theory of simplexes and barycentric coordinates allows for the transition to quadratic, cubic and n-dimensional interpolation using the same input base vectors (test signals) as with linear interpolation, and this makes it possible as will be shown below, if necessary, algorithmically and hardware-correct the interpolation method, i.e. automatically, if it is necessary to increase the accuracy of processing, from linear interpolation to quadratic, cubic, or, if the measured multidimensional value changes in some sense smoothly, do the opposite - move from quadratic, cubic interpolation to linear.

Let $\{\vec{x}^v\}, v = \overline{1, n}$ multidimensional simplicial test signals (used in the linear interpolation method).

$$\vec{x}^{1T}(1, 0, \dots, 0, \dots, 0); \dots; \vec{x}^{sT}(0, 0, \dots, 1, \dots, 0); \dots; \vec{x}^{nT}(0, 0, \dots, 1), \quad (2)$$

with corresponding one-dimensional outputs.

For the quadratic interpolation method, we will generate additional multidimensional signals $\vec{x}^{v,s}$, which are the midpoints of the simplex edges connecting the v-th vertex to the s-th vertex. These additional multidimensional signals are determined by the simple relation

$$\vec{x}^{v,s} = \vec{x}^{s,v} = \frac{1}{2}(\vec{x}^v + \vec{x}^s) = \frac{1}{2}(\vec{x}^s + \vec{x}^v) \quad (3)$$

It is easy to show that in the n-dimensional case the number of input multidimensional test signals for the quadratic interpolation method will be equal to

$$N_2 = C_n^0 + 2C_n^1 + C_{n+1}^2 = \frac{(n+1)(n+2)}{2} \quad (4)$$

For example, for $n = 1$, $N_2 = 3$, for $n = 2$, $N_2 = 6$, for $n = 3$, $N_2 = 10$.

Note that the number of all coefficients of the quadratic form from n variables is also equal to the number of points $N_2 = \frac{(n+1)(n+2)}{2}$.

We will obtain the quadratic interpolation polynomial from the following basic system of equations by coincidence at the nodes

$$\begin{cases} Q(\vec{x}^v) = u_v = u(\vec{x}^v); & v = \overline{0, n} \\ Q(\vec{x}^{v,s}) = u_{v,s} = u(\vec{x}^{v,s}); & v = \overline{0, n}; v \neq s \end{cases} \quad (5)$$

where $u_{v,s} = u(\vec{x}^{v,s})$ — output signal (response) to input signal $\vec{x}^{v,s}$.

Naturally,

$$\begin{aligned} Q(\vec{x}^{v,s}) &= Q(\vec{x}^{s,v}) = u_{v,s} = u_{s,v} \\ &= u(\vec{x}^{v,s}) = u(\vec{x}^{s,v}), \end{aligned}$$

because $\vec{x}^{v,s} = \vec{x}^{s,v}$.

In this case, the number of equations in (5) will be equal to $n + 1 + \frac{(n+1)^2 - (n+1)}{2} = \frac{(n+1)(n+2)}{2} = N_2$, i.e., coincides with the number of coefficients of the quadratic form of n variables.

To determine the interpolating quadratic form of n variables in the local system of barycentric coordinates, we prove the following theorem.

Theorem 1. The interpolating quadratic form of n variables is as follows

$$Q(\vec{x}) \equiv \sum_{v=0}^n [2\lambda_v^2(\vec{x}) - \lambda_v(\vec{x})] u_v + 2 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\vec{x}) \lambda_s(\vec{x}) u_{v,s}, \quad (6)$$

where $\lambda_v(x)$ - barycentric coordinates of point x relative to the vertices of the simplex x_v , defined by the relations

$$x = \sum_{s=0}^n \lambda_s(\vec{x}) \vec{x}^s, \quad (7)$$

$$\sum_{s=0}^n \lambda_s(\vec{x}) = 1, \quad (8)$$

$$\lambda_s = \sum_{i=1}^n z_{is} x_i + z_{os}; s = \overline{0, n} \quad (9)$$

Proof:

Since $\lambda_v(x)$ are linear functions of \vec{x} according to (9), and they enter into relation (1) only quadratically and linearly, then first of all $Q(x)$ is a quadratic

polynomial with respect to the n -dimensional vector \vec{x} .

Now all that remains is to show that $Q(x)$, given by relation (6), is an interpolation quadratic polynomial, i.e. satisfies the system of relations

$$\begin{aligned} L(\vec{x}) &= \sum_{v=0}^n \lambda_v(\vec{x}) u(\vec{x}) = \sum_{v=0}^n \lambda_v(\vec{x}) \left[u(\vec{x}) + \right. \\ &+ (\nabla u(\vec{x}), \vec{x}^v - \vec{x}) + \frac{1}{2} [H(y^v)(\vec{x}^v - \vec{x}), \vec{x}^v - \vec{x}] \left. \right] = \\ &u(\vec{x}) \sum_{v=0}^n \lambda_v(\vec{x}) + \sum_{v=0}^n (\nabla u(\vec{x}), \vec{x}^v - \vec{x}) \lambda_v(\vec{x}) + \\ &+ \frac{1}{2} \sum_{v=0}^n [H(y^v)(\vec{x}^v - \vec{x}), \vec{x}^v - \vec{x}] \lambda_v(\vec{x}). \end{aligned} \quad (10)$$

Due to the fact that $\lambda_v(\vec{x}^v) = \delta_{vk}$, then $Q(\vec{x}^k) = u_k = u(\vec{x}^k)$; $k = \overline{0, n}$, where δ_{vk} is the Kronecker symbol (the first from relation (5)). If $u(x)$ is not a linear mapping from the n -dimensional space E_l to the number axis \vec{z} and \vec{w} and $w \vec{}$ are two n -dimensional vectors ($\vec{z} \in E_n$, $\vec{w} \in E_n$), then the following relation holds:

$$\begin{aligned} u(\vec{w}) &= u(\vec{z}) + \sum_{s=1}^k \frac{1}{s!} D^s u(\vec{z})(\vec{w} - \vec{z})^s + \\ &+ \frac{1}{k+1} D^{k+1} [\vec{z} + \theta(\vec{w} - \vec{z})](\vec{w} - \vec{z})^{k+1}, \end{aligned} \quad (11)$$

where $D^s u(\vec{z})$ - are the s -th multidimensional derivatives of the nonlinear mapping $u(\vec{z})$, which are symmetric s -linear mappings, $(\vec{w} - \vec{z})^s$ - s is the short product of the vector $(\vec{w} - \vec{z})$ - onto itself for all $s = \overline{1, k+1}$, θ is a real number satisfying the relation $0 < \theta < 1$.

Based on all of the above, we can estimate the error of the quadratic interpolation method.

Theorem 2. For any point $\vec{x} \in E_n$ the following relations hold true:

$$\begin{aligned} Q(\vec{x}) - u(\vec{x}) &= \frac{1}{6} \left\{ \sum_{v=0}^n D^3 u(y^v)(\vec{x}^v - \right. \\ &- \vec{x})^3 [2\lambda_v^2(\vec{x}) - \lambda_v(\vec{x})] + \\ &\left. \sum_{\substack{v,s=0 \\ v \neq s}}^n D^3 u(y^{v,s})(\vec{x}^{v,s} - - \vec{x})^s \lambda_v(\vec{x}) \lambda_s(\vec{x}) \right\}, \end{aligned} \quad (12)$$

$$\begin{aligned} \nabla Q(\vec{x}) - \nabla Q(\vec{x}) &= \frac{1}{6} \left\{ \sum_{v=0}^n D^3 u(y^v)(\vec{x}^v - \right. \\ &- \vec{x})^3 [4\lambda_v(\vec{x}) - 1] \nabla \lambda_v(\vec{x}) + \\ &\left. 4 \sum_{\substack{v,s=0 \\ v \neq s}}^n D^3 u(y^{v,s})(\vec{x}^{v,s} - \vec{x})^3 \lambda_v(\vec{x}) \nabla \lambda_s(\vec{x}) \right\}. \end{aligned} \quad (13)$$

Proof:

Using the Taylor series expansion up to the 3rd term inclusive in the multidimensional case, we have

$$\begin{aligned} u(\vec{x}^v) &= u(\vec{x}^v) + Du(y^v)(\vec{x}^v - \vec{x}) + \frac{1}{2} D^2 u(y^v)(\vec{x}^v - \vec{x})^2 \\ &+ \frac{1}{6} D^3 u(y^v)(\vec{x}^v - \vec{x})^3. \end{aligned} \quad (14)$$

$$\begin{aligned} u(\vec{x}^{v,s}) &= u(\vec{x}) + Du(y^{v,s})(\vec{x}^{v,s} - \vec{x}) + \\ &+ \frac{1}{2} D^2 u(y^{v,s})(\vec{x}^{v,s} - \vec{x})^2 + \frac{1}{6} D^3 u(y^{v,s}) \\ &(\vec{x}^{v,s} - \vec{x})^3. \end{aligned} \quad (15)$$

Since relation (6) is true, then substituting relations (14) and (15) into it, and also taking into account the following easily verified relation

$$\sum_{v=0}^n [2\lambda_v^2(\bar{x}) - \lambda_v(\bar{x})] + 2 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x})\lambda_s(\bar{x}) = 1 \quad (16)$$

and the fact that for $Du(\bar{x})$ there will be a factor

$$\begin{aligned} & \sum_{v=0}^n [2\lambda_v^2(\bar{x}) - \lambda_v(\bar{x})](\bar{x}^v - \bar{x}) + 2 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x})\lambda_s(\bar{x})(\bar{x}^{v,s} - \bar{x}) = \\ & = 2 \left(\sum_{s=0}^n \lambda_s(\bar{x}) \right) \left[\sum_{v=0}^n \lambda_v(\bar{x})(\bar{x}^v - \bar{x}) - \sum_{s=0}^n \lambda_{vs}(\bar{x})(\bar{x}^s - \bar{x}) \right] \end{aligned}$$

equal to 0 in connection with (7), we obtain that in the Taylor series expansion the second-order term will have the form

$$\begin{aligned} & \frac{1}{2} \sum_{v=0}^n [2\lambda_v^2(\bar{x}) - \lambda_v(\bar{x})] D^2 u(\bar{x})(\bar{x}^v - \bar{x}) + \\ & + \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x})\lambda_s(\bar{x}) D^2 u(\bar{x})(\bar{x}^{v,s} - \bar{x}) \equiv \\ & \equiv \frac{1}{2} D^2 u(\bar{x}) \left[\sum_{v=0}^n \lambda_v(\bar{x})(\bar{x}^v - \bar{x})^2 + \right. \\ & \left. + \frac{1}{2} D^2 u(\bar{x}) \sum_{v=0}^n [\lambda_v(\bar{x})(\bar{x}^v - \bar{x})(\sum_{s=0}^n \lambda_s(\bar{x}) - 1)(\bar{x}^s - \bar{x})] \right]. \end{aligned} \quad (17)$$

Due to the existence of relations (7) and (8), this term is also equal to 0, which proves the relation of theorem (12). To prove the second part of the theorem, i.e. relation (13) we differentiate the expression for the quadratic interpolation polynomial (6).

$$\begin{aligned} DQ(\bar{x}) &= \sum [4\lambda_v(\bar{x}) - 1] D\lambda_v(\bar{x})u(\bar{x}^v) + \\ & + 4 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x}) D\lambda_s(\bar{x})u(\bar{x}^{v,s}) \end{aligned} \quad (18)$$

Applying to $u(\bar{x}^v)$ and $u(\bar{x}^{v,s})$ the Taylor series expansion up to the 3rd term inclusive in relation (18), for $u(\bar{x})$ we obtain the following factor

$$\begin{aligned} & \sum_{v=0}^n [4\lambda_v(\bar{x}) - 1] D\lambda_v(\bar{x})u(\bar{x}) + 4 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x}) D\lambda_s(\bar{x}) = \\ & = 4 \sum_{v=0}^n \lambda_v(\bar{x}) D\lambda_v(\bar{x}) - \sum_{v=0}^n D\lambda_v(\bar{x}) + 4 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x}) D\lambda_s(\bar{x}). \end{aligned}$$

Since $\sum_{s=0}^n \nabla \lambda_s = 0$ it equals to 0

$$\begin{aligned} & 4 \sum_{v=0}^n \lambda_v(\bar{x}) D\lambda_v(\bar{x}) + 4 \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v(\bar{x}) D\lambda_s(\bar{x}) = \\ & = 4 \sum_{v=0}^n \lambda_v(\bar{x}) \left[\sum_{v=0}^n D\lambda_v(\bar{x}) + \sum_{v=0}^n D\lambda_s(\bar{x}) \right] = 0 \end{aligned}$$

In a similar way, it can be shown that the factors of $Du(x)$ and $D^2u(x)$ will be equal to 0, which indicates the validity of estimate (15) and completes the proof of the theorem.

When considering the cubic interpolation method, it is assumed that test signals in the form of vertices of the simplex $\{\bar{x}^v, v = \overline{0, n}\}$ are supplied to the input of the device as before, and auxiliary signals are generated using special computing devices.

$$\begin{aligned} & \{\bar{x}^{v,s,k}; v, s, k = \overline{0, n}; v \neq s \neq k\}, \\ & \bar{x}^{v,s,k} = \frac{1}{3} (\bar{x}^v + \bar{x}^s + \bar{x}^k). \end{aligned} \quad (19)$$

To attract information about the dynamics of the controlled process, it is advisable to consider a modified scheme for interpolating multidimensional functions.

This can be implemented in hardware by using multidimensional differentiators in devices and systems. In this case, the modified method of multidimensional cubic interpolation is carried out based on the following necessary relations.

$$\begin{cases} k(\bar{x}^v) = u_v = u(\bar{x}^v); v = \overline{0, n} \\ \nabla k(\bar{x}^v) = Dk(\bar{x}^v) = Du(\bar{x}^v) = \nabla u(\bar{x}^v); v = \overline{0, n} \\ k(\bar{x}^{v,s,k}) = u_{vks} = u(\bar{x}^{v,s,k}); v, s, k = \overline{0, n}; v \neq s \neq k \end{cases} \quad (20)$$

Involving the differential relation in (20) instead of the second relation in (19) saves us from additional test changes in the middles of the faces, but artificially due to multidimensional differentiation $Dk(\bar{x}^v) = \nabla u(\bar{x}^v)$ increases the information content (for example, the number of ratios used increases by 2 times).

The latter makes it possible to significantly increase the accuracy of computational processes when processing information about vector quantities. In the proposed cubic interpolation method, the number of informative relations is $[c_n^0 + 3c_n^1 + 3c_n^2 + c_n^3]$. The form of the cubic interpolation polynomial is determined by the following theorem.

Theorem 3. In terms of barycentric coordinates, the form of a cubic multidimensional interpolation polynomial satisfying relation (20) is defined as follows

$$\begin{aligned} k(\bar{x}) &= \sum_{v=0}^n [-2\lambda_v^3(\bar{x}) + 3\lambda_v^2(\bar{x})] u_v + \\ & \frac{1}{6} \sum_{v,s,k=0}^n \lambda_v(\bar{x})\lambda_s(\bar{x})\lambda_k(\bar{x}) \\ & [27 + u_{vsk} - 7(u_v + u_s + u_k)] + \\ & + \sum_{\substack{v,s=0 \\ v \neq s}}^n \lambda_v^2(\bar{x})\lambda_s(\bar{x}) [\nabla u(\bar{x}^v), \bar{x}^s - \bar{x}^v] - \\ & - \sum_{\substack{v,s,k=0 \\ v \neq s \neq k}}^n \lambda_v(\bar{x})\lambda_s(\bar{x})\lambda_k(\bar{x}) [\nabla u(\bar{x}^v), \bar{x}^s - \bar{x}^v] \end{aligned} \quad (21)$$

Proof:

It is easy to see that the first and third relations in (20) for the interpolation polynomial (21) are satisfied.

To check the validity of the second relation in (20), we differentiate the interpolating cubic polynomial of many variables (21) and obtain:

$$\nabla k(\bar{x}^v) = \sum_{\substack{v,s=0 \\ v \neq s}}^n (\nabla k(\bar{x}^v), \bar{x}^s - \bar{x}^v) \nabla \lambda_s(\bar{x}^v) \quad (22)$$

The rate of change of an arbitrary v – th barycentric coordinate $\lambda_v(\bar{x})$; $v = \overline{0, n}$ in the direction $\bar{x}^s - \bar{x}$; $s = \overline{0, n}$, characterized by the relation

$$[\nabla \lambda_v(\bar{x}), \bar{x}^s - \bar{x}] = \delta_{v,s} - \lambda_v(\bar{x}) \quad (23)$$

Using (23) one can easily obtain the equality $\nabla k(\bar{x}^v) = \nabla u(\bar{x}^v)$ which, coinciding with the second relation (20), proves the theorem.

Estimating the error of the cubic interpolation method (20) using multidimensional differentiation is carried out similarly to estimating the error of the quadratic interpolation method.

Using the method of barycentric coordinates, it is possible to obtain algorithms and error estimates of algorithms for multidimensional interpolation of any degree. In this case, multidimensional differentiation can be used in various combinations. If the increase in the amount of information is insignificant (for example, in the case of interpolation by polynomials of low degrees), then you can use the classical scheme and do without differentiation.

By establishing a criterion for assessing the error of a vector quantity, you can change the type of multidimensional interpolation in the control process, i.e. switch to spline approximation and make the multiparameter device or system adaptive.

3 CONCLUSIONS

Thus, for mathematical description of characteristics of multiparameter converters based on specified experimental data, it is advisable to use methods of linear, quadratic, cubic and n -dimensional interpolation based on the theory of barycentric coordinates and the theory of simplexes. Comparison of methods for approximating functions of transformation of multiparameter converters allows choosing the optimal method for a specific task - minimization of the structural diagram of the computing device. This, in turn, allows more accurate determination of metrological characteristics of measuring converters, to propose effective methods and means for processing primary measurement information. The obtained results will allow, based on

the use of computer modeling tools using the capabilities of modern computing systems, to implement effective model support for the development of new and improvement of existing methods for processing signals of primary converters and systems. This will provide an opportunity to achieve a new level of technical and economic efficiency of measuring instruments. The results of the work can also find application in various industries where there is a need to create and improve measuring instruments.

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