Using the Program Evaluation and Review Technique and the Critical Path Method to Help Project Managers

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Abstract: This study examines two fundamental project management techniques: the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT). CPM is a deterministic approach designed for repetitive projects with well-defined task durations, enabling the identification of critical activities that directly determine project completion time. PERT, in contrast, is a probabilistic method suited for projects with uncertain task durations, incorporating optimistic, most likely, and pessimistic time estimates to account for variability and risk. Through numerical applications and comparative analysis, the study highlights the methodological differences, computational procedures, and practical implications of each technique. The findings demonstrate that CPM is effective for structured, predictable projects, while PERT provides superior flexibility under conditions of uncertainty. By evaluating the strengths and limitations of both methods, this work offers practical guidance for project managers in selecting appropriate scheduling tools based on project complexity, uncertainty levels, and resource availability, ultimately supporting efficient project planning and timely completion.

1 INTRODUCTION

Networks can be used to describe and assess the success of most project kinds, as well as to identify specific outcomes like cutting down on project completion time and determining which tasks should be expedited to shorten the project's overall duration. As a result, the project's longest path from start to finish determines the minimum overall project time. In the 1950s [1], a helpful tool called the critical path technique emerged to assist managers in organizing and scheduling major projects. This technique complements the Gantt chart and can be used in place of it. In 1959 [2], Kelly and Walker drew attention to the application of (PERT) and (CPM) in project management in a range of industries. PERT and CPM differ in that PERT is mainly utilized in situations where the time needed to complete each task is uncertain, but the CPM approach is based on project execution on the understanding and expertise of previous projects [3], [4]. The primary market for the CPM was the construction sector. The use of CPM is a part of total quality management in health services, and it was tested in a pilot program [5]. A deterministic term analysis of a duration project can be performed with (CPM) [6]. Total quality management aims to improve an organization's culture, practices, products, and services to satisfy consumers. In project management studies, planning is a crucial and challenging topic [7]. The complexity and dynamic nature of the undertaking both contribute to its difficulties. Plans are constantly being revised before and during their execution [8]. Project management methods, such as CPM, analyze project durations deterministically, finding the minimum intended duration [9]. Some techniques, like PERT, use data values for specific tasks as arbitrary variables, such as hopeful (shortest), most probable (middle), and pessimistic (longest) estimation times [10]. These methods help manage and oversee projects based on network dependencies and changes in project conditions [11]. The construction industry has been the primary user of CPM in the past [12]. CPM is among the different methods for carrying out project planning with PERT. CPM is utilized in projects consisting of multiple discrete tasks [13]. In (2022) [14], Bashiru and Douglas showed that CPM allows project managers to schedule work, track project objectives, and make necessary adjustments; it serves as a mirror for the project, enabling resource planning and schedule preparation, and ensuring the project is executed correctly [15].

By determining the critical path a set of tasks that must be finished promptly in order for the project to be finished on time the critical path approach is a strategy for planning and organizing project schedules [16]. When the oldest and most recent times of an action are the same, it is said to be essential [17]. This work is concerned with presenting two useful techniques for project managers in scheduling, planning and determining the maximum possible time to complete projects, which is defined by the critical path, and from it determining the critical activities in the project, as well as some numerical applications of each technique and the most prominent differences between the two techniques.

2 SOME IMPORTANT DEFINITIONS

Network [18]: An interconnected geometric representation that includes two types of data: events and activities. It begins with one initial event called the beginning and ends with one final event called the end. Each activity contains a time that represents the time required to complete the activity

Activity [19]: is any straight or curved arrow that is not broken on the network that connects two events and has a beginning and an end. The activity takes time and resources, and no activity can be started until all activities are completed. The previous one.

Event [20]: In the network, an event is represented by a numbered circle, or it can be a numbered square or triangle that represents the beginning and end of the activity in a sequential manner and does not take time or resources. There is a head and a tail of the event, and there are types of events , merged events and burst events. Generally, the head events have a higher value than the tail events.

Predecessor event and successor event [7]: An event is predecessor if it comes directly before an event. An event is successor event if it follows an event directly. So, any event may be a predecessor event for an event and successor event for another events.

Critical path [21]: The bigger path connecting the network of project from beginning to end is made up of the network's critical activities and summation of all critical activities.

3 CRITICAL PATH METHOD (CPM)

It is a commonly recognized tool utilized to schedule and manage tasks in complex construction networks. CPM enables project managers to identify the sequence of tasks that determine the project's overall duration, known as the critical path [22]. After representing the project in the form of a network, the next step is calculating the total time to complete the project by CPM.

3.1 Forward Pass (Earliest Time)

The computations start with Activity 1 and advance recursively to Activity n, including the earliest start time (ES) and earliest finished time (EF).

In the first step:

$$E_1 = 0. \tag{1}$$

Second step: let i, j are two Activities, A, B There are two activities and D inducator to the duration between two Activities then

$$ESB = EFA + Di, j \,. \tag{2}$$

General Step j. Given that events $A, B..and\ K$are connected directly to the event j by incoming activities $(A, j), (B, j)..and\ (K, j)$ and that the earliest occurrence times of events $A, B, .$ and K have already been computed, then the earliest occurrence time of Activity j is

$$E_j = \max\ \{EA + DA, j\ , EB, j + DB, j\ , \ldots . EK + DK, j\}. \tag{3}$$

The forward pass [23] is complete when En at event n has been finished, Ej is the longest path (duration) to the event j.

3.2 Backward Pass (Latest Times)

This computations start at node n and ends at node 1 and including latest start time (LS) and latest finshed time (LF).

Firstly, for any network then:

$$L = E\ , \tag{4}$$

To indicate that latest time of the last event the same value of the total duration of the project.

Second step: Let i, j are two events and A, B are two activities between i, j then:

$$LSB = LFA - Di, j, \tag{5}$$

General Step j: Given that events $A, B, \ldots.$ and $K$are connected directly to the event j by outgoing activities $(j, A), (j, B) \ldots.. and\ (j, K)$ and that the latest events $A, B \ldots.$ and K have computed the latest time is:

$$Lj = \min\{LA - Dj, A\ , LB - Dj, B \ldots. LK - Dj, K\ \} \quad (6)$$

The backward pass (Latest Times, L) [24], [25] ends at event 1 with:

$$L1 = 0. \quad (7)$$

By above calculations, an activity $A = (i, j)$ Then the critical if it satisfies three conditions.

$$\text{*-}Ei = Li \quad , \text{**-}Ej = Lj\ , \quad \text{***-}Ej - Li = Di, j. \quad (8)$$

Any activity that does not meet all three requirements is considered noncritical [14].

4 THE PROGRAMEVALUATION AND REVIEW TECHNIQUE

PERT is a statistical technique used in project management that was created to describe and analyze the activities required to finish a project, particularly the amount of time needed to do each task and determine the minimal amount of time needed to finish the project as a whole [26]. The three estimates are plotted on a graph, with the probability distribution closely associated with the Beta distribution curve, which is unmoral and not symmetric [27]. Three primary sorts of estimations can be produced using probabilistic time estimation [28], [29]:

1) A pessimistic time estimate (T_p).
2) A hopeful time estimate (T_h).
3) A most probable time estimate (T_m).

The hopeful time estimate assumes a task can be completed quickly, while the pessimistic estimate anticipates unexpected issues, leading to increased time consumption. The most likely estimate falls between these two as calculated in Tables 1 and 2.

$$Tp \geq Tm \geq Th\ . \quad (11)$$

The most probable time estimate should be given a weight of 4, and the hopeful and pessimistic time predictions should be given a weight of 1. A time estimate (Te)is obtained as the weighted (mean) The following is the average of these estimations:

$$Te = \frac{Tp + 4Tm + Th}{6}, \quad (12)$$

and Variance (σ^2)

$$\sigma^2 = \left(\frac{Tp - Th}{6}\right)^2, \quad (13)$$

where the root of variance is known as the Standard deviation (σ) [28]

$$\sigma = \left(\frac{Tp\_Th}{6}\right), \quad (14)$$

$$\sum T\ critical = \textit{Calculation of the total sum of anticipated critical values}, \quad (15)$$

$$Vcritical = \textit{Calculation of all variances of critical activities}. \quad (16)$$

PERT uses simplifying plan updates during network modifications.

Problem 1. To find project time by using CPM for the following activity network as in Figure 1.

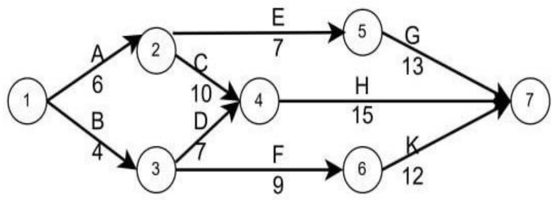

Figure 1: Network diagram.

Table 1:CPM applcation.

Activity	Duration (weeks)	ES	EF	LS	LF
A	6	0	6	6	0
B	4	0	4	9	0
C	10	6	16	16	6
D	7	4	16	16	9
E	7	6	13	18	6
F	9	4	13	19	9
G	13	13	31	31	18
H	15	16	31	31	16
K	12	13	31	31	19

The critical path is $A \rightarrow C \rightarrow H$ .Therefor, the project time complete = Critical path length = 31 Weeks

Problem 2: We will use PERT to find the time complete to the following data.

From the three time estimates T_p, T_m and T_h calculate T_e for each activity, show Table 3.

The four paths are as follows:

1) Path 1: $A \rightarrow C \rightarrow H$, Time path = 6+10+11 = 27weeks, Path 3: $A \rightarrow E \rightarrow G$, Time path = 6+7+7 = 20 weeks

2) Path 2: $B \rightarrow D \rightarrow H$,Time path = 4 + 7 + 10 = 21 weeks, Path 4: $B \rightarrow F \rightarrow K$,Time path = 4+7+5=16 weeks

From the four paths. Maximum of {27,21,20,16} = 27. Path 1 is the critical path, i.e., $B \rightarrow M \rightarrow N \rightarrow K$. And the critical activities are B, M, N and K. Project time Complete = Critical path length =34 , i.e $\sum Ŧ\, critical = 6 + 10 + 11 = 27\, weeks$ and the total variance of the critical path = Calculation of all variances of essential activities i.e. $Vcritical = 0.444 + 1.777$+5.444=$7.665$

The above problems show that the difference between PERT and CPM techniques is 4 weeks. Differences between PERT and CPM techniques in scheduling and planning projects which listed in Table 4.

Table 2: PERT application.

Activity	Predecessor Activity	Duration (weeks)	Pessimistic time estimate (Ŧp)	Most probable time estimate (Ŧm)	Hopeful time estimate (Ŧh)
A	-	6	9	6	3
B	-	4	6	4	2
C	A	10	7	5	3
D	B	7	18	5	4
E	A	7	9	8	3
F	B	9	16	8	6
G	E	13	8	7	5
H	D,C	15	3	2	1
K	F	12	8	5	2

Table 3: PERT Calculation results and variance (σ ^{2}).

Activity	Duration (weeks)	T_p	$4T_m$	T_h	$T_e = \frac{T_p + 4T_m + T_h}{6}$	$σ^2 = \left(\frac{T_p - T_h}{6}\right)2$
A	6	9	24	3	6	1
B	4	6	16	2	4	0.444
C	10	14	40	6	10	1.777
D	8	18	20	4	7	5.444
E	7	28	12	2	7	18.777
F	9	16	32	6	9	2.777
G	8	10	28	4	7	1
H	12	20	40	6	11	5.444
K	5	8	20	2	5	1

Table 4: Comparison between CPM and PERT.

PERT	CPM
1. PERT is used to manage uncertain project activities; it uses a probabilistic model. 2. PERT is used where the time required to complete the activities is unknown. 3. The network is constructed based on the events. 4. Because jobs are non-repetitive, PERT has been given attention in areas like research and development programs. 5. Critical and non-critical activities are not differentiated 6. Management of Unpredictable Activities. 7. While notations like arrow diagram, node, task, float, etc. are always used in CPM, PERT uses notations like network, event activity, slack, etc.	1. CPM is a tool used to manage statistics well. It uses a deterministic model. 2. In CPM, there is just one estimate 3. CPM is used mainly for construction programs. 4. Critical and non-critical activities are differentiated 5. Management of predictable Activities 6. CPM notations like arrow diagram, event, function, slack, etc., are invariably used.

5 CONCLUSIONS

This work explores the use of PERT and CPM in project management. CPM is accurate for specific activities, while PERT works on uncertain projects by estimating three different periods. Combining both methods can improve planning and scheduling, reducing potential obstacles, which helps project managers to achieve the best goals within the project time frame. When using the CPM on a project containing nine activities, the total time was 31 weeks as a maximum to complete the project without delaying the delivery date, while when conducting the PERT method on the same project after entering the three most likely ominous and hopeful estimates, the total time was 27 weeks, a difference of 4 weeks from CPM. CPM calculates total time sequentially based on activities, while PERT prioritizes activities without strictly sequencing them. Both methods are successful in planning, scheduling, and managing large projects, and selecting the appropriate method depends on the project's complexity, level of uncertainty, available resources, and specific management goals to ensure optimal efficiency and timely completion.

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