

Using the Rooted Tree to Find the Optimal Solution for the Transportation Problems

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Abstract: Considering economic aspects of the transportation problem and the fact that it is a particular instance of linear programming problems that rely on figuring out the best way to distribute goods from supply centers to customers in a shorter amount of time or at a lower cost, as well as the growing importance of the globalization and rapid development era, resolving the transportation problem has become crucial in the field of operations research to enhance ways of bringing the product in the quickest amount of time or at the lowest cost from the original source to the customer. By developing a new algorithm to address transportation issues, including logistical issues for the supply of goods and their arrival at their destination, we offer a simplified, guaranteed, and less expensive approach than earlier approaches for businesses to deal with transportation problems, thereby improving efficiency, reducing operational risks, and significantly increasing overall competitive advantage for companies.

1 INTRODUCTION

Transportation problems (TP) of the repeatedly used applications of programming with linear techniques is for mutating and solvin transportation problems as a Lpp. in this problem, the main target is obtain an optimal schedule for shipping the commodity while satisfy in the demands in every destination, French mathematician Gaspard Monge in 1781, proposed a mathematical model that in volves sealing and transporting soil between different locations at the lowest possible cost. This achieved through aheavy cooperation with Napoleon Bonaparte's army that worked to build forts and military ways. The American mathematician, Frank L. Hitchcock, proposed the TP in 1941 [1], and A.N. Tolstoi [2] published his Monge's problem solution which represents atheoretical foundation for solving TP. in1947, Tjalling C. Koopmans, offered his paper to solve TP [3]. the tow mentioned studies are the main achievements in the advancement of the different techniques for solving the transportation model. also, in 1949 T.C. Koopmans developed the transportation model and G. B. Dantzing towards the for mulation and solution of linear programming problem in 1951 [4] with using simplex method for getting optimal solution. Bazarra, Jarvis and Sherali defined

the linear programming problems in 1990 [5]. However, it includes aconsiderable volume of variables and restrictions, and solve them by simplex technique requires of effort and a protracted time. Abdul Staar Soomro et.al. suggested an adjustment vogel's approximate method for solving transportation problem in 2015 [6]. Many researchers have presented other methods to get (IBFS) to the transportation problems. Lakh veer Kaur and others discussed now to improve the method of maximum difference for finding (IBFS) for the TP in 2018 [7]. In arelated context, S.C.Zelibe and C.P.Ugwuanyi developed novel remedies for the problem of transportation, 2019 [8]. In 2020 Al-Saeedi 1 proposed a technique for finding objective solutions for transportation types. Subsequently, Noor proposed a technique for finding the optimal or near-optimal solution [9]. In this work we present a new approach to find the optimal solution for the transportation problems which minimize is cost by using an ordered tree, where a tree is an directed graph in which any two vertices are connected by exactly one path, or equivalently a connected acyclic directed graph [10], so we sought to provide the best results that showed the efficiency of the new method by comparing their solution to the old ones (NWCM, LCM and VAM) and the method Saeedi 1. We used

the Matlab 2013 program to calculate the optimal solution; it is given to the optimal solution.

2 SOME BASIC CONCEPTS OF TRANSPORTATION PROBLEMS TP AND GRAPH THEORY (GT)

2.1 Transportation Model

In TP, there exist m origins of supply $S_1, S_2, \dots, S_m$ and n destinations of demand $D_1, D_2, \dots, D_n$, each one is specially represented. The brackets symbolize the tracks connecting origins and destinations. Bracket (i, j) connection from origin i ($i = 1, 2, \dots, m$) into destination j ($j = 1, 2, 3, \dots, n$), where c_{ij} represents the transportation cost for each unit of product, while x_{ij} is the charged quantity. The available units at supply in origin i are represented by a_i , while the available units at request in destination j are represented by b_j .

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \text{ (gross cost)} \quad (1)$$

Subject to

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, \dots, m \text{ (supply restrictions)} \quad (2)$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, 2, \dots, n \text{ (demand restrictions)} \quad (3)$$

2.1.1 Representation of Transportation Problems

TP represent a particular model of linear programming problems (LPP). The transportation problem (Fig. 1) is shipping different quantities of homogeneous goods from various sources (e.g., factories) to various destinations (e.g., warehouses) so that the total transportation cost (or time) is minimized.

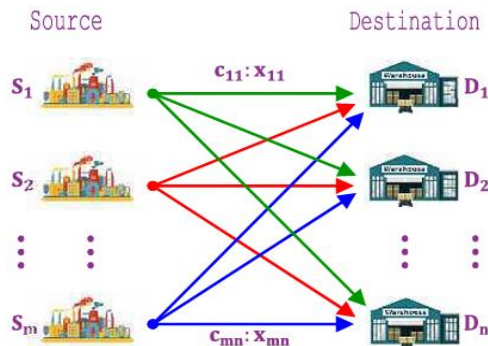


Figure 1: The graph structure of TP.

2.1.2 Schedule of Transportation

The TP is represented in a distinctive table (Fig. 2) that employs the mathematical model for the TP linear programming. A tabular format can be used to represent the transportation problem model with all related parameters. The supply (a_i Availability), visualized in the transportation schedule, is shown in every source in the right column, while the destination requirement (b_j) is found in every cell in the bottom row. The cost of oneness transfer (c_{ij}) is located in the top corner of the table, right Side, while the shipped matter quantity (x_{ij}) is represented in the cell center [11].

Sources (i)	Destinations (j)				Supply (a_i)
	D_1	D_2	$\dots$	D_n	
S_1	c_{11} x_{11}	c_{12} x_{12}	$\dots$	c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}	$\dots$	c_{2n} x_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	c_{ij} x_{ij}	$\vdots$	$\vdots$
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}	$\dots$	c_{mn} x_{mn}	a_m
Demand (b_j)	b_1	b_2	$\dots$	b_n	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

Figure 2: Schedule of TP.

2.1.3 Definition

2.1.3.1 Feasible Solution (FS)

A feasible solution to transportation problems is a collection of non-negative assigned quantities $x_{ij} \geq 0$ that cater for restrictions (supply constraints and demand restrictions in transportation problems) [12].

2.1.3.2 Basic Feasible Solution (BFS)

We say that a feasible solution of transportation problems is a basic feasible solution (BFS) when it includes no more than $(m \cdot n - 1)$ positive assigned quantities, where m stands for the number of rows and n stands for the number of columns of TP [12], [13].

3 GENERAL PROCEDURE TO SOLVE A TRANSPORTATION PROBLEM

The solution of a TP typically follows a systematic procedure designed to ensure both feasibility and optimality. The general approach consists of the following main steps:

- 1) Mathematical formulation of the table of TP.
- 2) Find an initial Basic Feasible Solution (BFS).
- 3) Modified the initial basic feasible solution (IBFS) obtained in Phase 2 to find the optimal solution.

3.1 Types of Transportation Problems

3.1.1 Balanced TP

The TP is called balanced when the quantum of products in the apportionment pivot is equivalent to the quantum of products desired by the request pivot i.e., $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$.

3.1.2 Unbalanced TP

The TP is called unbalanced when the quantum of products ready in the apportionment pivot is not equivalent to the quantum of products desired by the request pivot i.e. $\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$.

3.2 Methods for Finding IBFS of TP

The initial solution that we obtain from the traditional solution approaches or the new solution approaches must contain the following properties:

- It should be feasible.
- It must satisfy the non-negativity.
- It should be basic.
- Some of the classic methods used to get the first solution for TP are: North-West Corner Method (NWCN).
- Least Cost Method (LCM).
- Vogel's Approximation Method (VAM).

3.3 Methods of Finding Optimal Solution

The moment of determining the initial basic feasible solution will lead to performing the optimality test in an effort to see whether the obtained feasible solution is optimal or not. This test can be conducted through applying the following methods:

- 1) Stepping Stone Method (SSM).
- 2) Modified Distribution (MODI) Method.

3.4 Graph Theory

Graph theory is a branch of discrete mathematics that studies mathematical structures known as graphs. A graph is formally defined as an ordered pair

$G = (V, E)$ where V is the set of vertices, and E is the set of edges connecting pairs of vertices. Graph theory models binary relationships and solves optimization problems involving networks and distributional problems [10].

3.4.1 Tree

A tree is a connected, cycle-free undirected graph, meaning that there is a single path between any pair of nodes. Thus, if a tree $G = (V, E)$ contains n nodes, then the number of edges $|E|$ is $n - 1$. Trees represent hierarchical structures, making them an essential tool in many optimization and algorithmic analysis applications [14].

3.4.2 Rooted Tree

A rooted tree is one in which one node is identified as the root, forcing the other nodes into a hierarchical order. Every node, with the exception of the root, has a distinct path to the root. A node's "children" are the nodes that come right after it, while its "parent" is the node that comes before it. Numerous methods and mathematical models used in optimization are based on this hierarchical structure [15].

3.4.3 Tree in Graph

In graph theory, a "tree in a graph" is defined as a subgraph consisting of a set of vertices and edges of the original graph, which is connected and acyclic. In other words, if the graph $G = (V, E)$ and a tree $T = (V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$, then T is a tree in G if the following conditions are met [10], [16]:

- 1) Connected. A path exists between any pair of vertices in T .
- 2) Cycle-free. No route can begin and finish at the same vertex without repeating edges.

When T includes all vertices V in the original graph, this subgraph is called a universal tree.

3.4.4 Definitions

Definition 1. An m -ary tree ($m \geq 2$) is a rooted tree in which every vertex has m or fewer children [15] as in Figure 3.

Definition 2. A complete m -ary tree is an m -ary tree in which every internal vertex has exactly m children and all leaves have the same depth, as shown in Figure 3 [15].

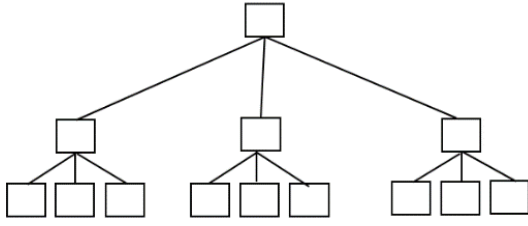


Figure 3: Complete 3-ary tree.

4 THE PROPOSED ALGORITHM

We will discuss the algorithm we proposed to find the optimal solution (OS) directly. This algorithm replaces the existing IBFS and OB in sub-sections 3.2 and 3.3 by relying on the rooted tree.

4.1 Algorithm Overview

This subsection describes the overall concept and purpose of the proposed method. The algorithm aims to optimize transportation costs by constructing a rooted tree instead of relying on traditional methods such as IBFS (Initial Basic Feasible Solution) and OB (Optimality by MODI or SSM). The rooted tree mechanism allows systematic allocation of supply and demand across the cost matrix, forming a basis for cost minimization.

4.2 Algorithm Procedure

The proposed algorithm follows a systematic approach to solve the transportation problem through the following steps:

Step 1. We start with the cell located at the North West corner, which takes its share from the allocation between supply and demand.

Step 2. From this cell, we begin forming a tree extension, taking the cells in the second row, with each cell taking its share between supply and demand, starting from the first cell.

Step 3. We take the branches of the cells that branched from the cell in the second row and continue the tree extension for the remaining cells until the last cell in the next row.

Step 4. We repeat the allocation for the cells so that each cell takes its share between supply and demand until the tree branching is complete.

Step 5. Repeat the above steps for each cell of the cost matrix and calculate the cost of the transportation problem.

Step 6. We choose the lowest cost for the transportation problem.

Note. We get the lowest cost from step 6 of the proposed algorithm, which matches the optimal solution (OS) after comparing it with SSM and MODI.

5 NUMERICAL EXAMPLES

We will provide illustrative examples of the proposed algorithm. The input data for the transportation problem are summarized in Figure 4.

Source	Destination			Supply
	D_1	D_2	D_3	
S_1	1	2	2	15
S_2	6	4	5	35
S_3	3	2	9	25
Demand	23	33	19	75

Figure 4: The transportation problems.

The following numerical example demonstrates the application of our algorithm to this dataset:

- 1) In Figure 5, we start from the first row and allocate the cell (northwest) until we complete allocating the elements of the first row.

Based on the first cell, we calculate the cost as

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 x_{ij} c_{ij}.$$

$$Z = (15*1) + (8*6) + (27*4) + (6*2) + (19*9) = 354 \text{ units.}$$

- 2) We repeat the algorithm for the first row by allocating the second cell, as in the following Figure 6.

Based on the second cell, we calculate the cost as

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 x_{ij} c_{ij}.$$

$$Z = (15*2) + (23*6) + (12*4) + (6*2) + (19*9) = 399 \text{ units.}$$

- 3) We repeat the algorithm for the first row by allocating the third cell, as shown in Figure 7.

Based on the third cell, we calculate the cost as

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 x_{ij} c_{ij}.$$

$$Z = (15*2) + (23*6) + (12*4) + (21*2) + (4*9) = 294 \text{ units.}$$

Now, we replaced the third row with the second row while maintaining the allocation for each column and row. By repeating the proposed algorithm on each cell, we will be able to calculate the lowest cost of the cost matrix that matches the optimal solution based on OS, as in the following Figures 8, 9, and 10.

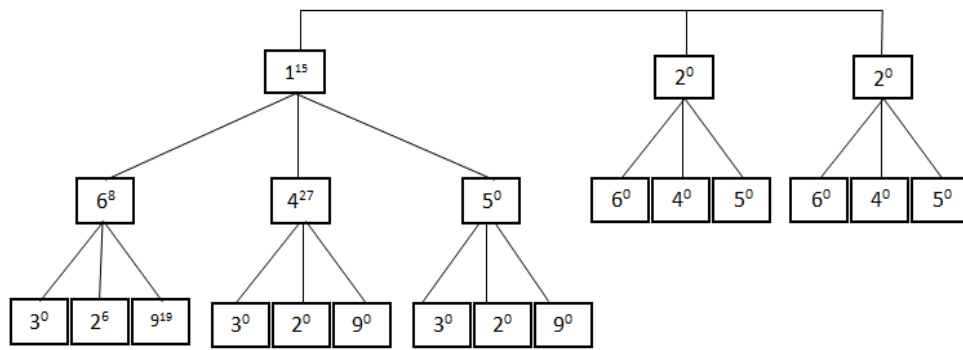


Figure 5: The tree of the first row rooted tree by c_{11} .

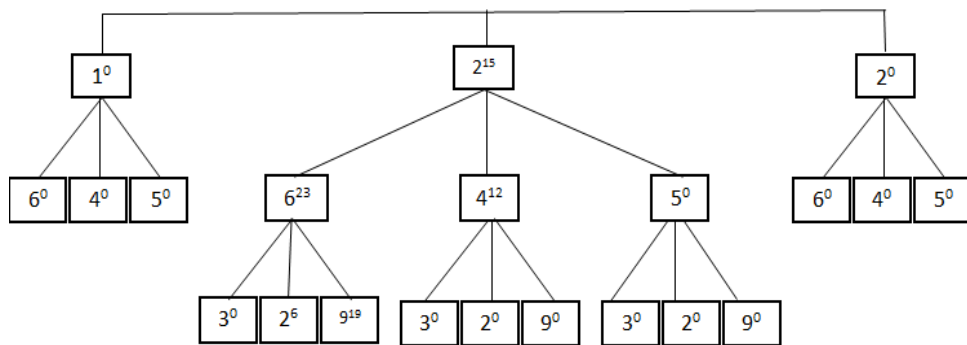


Figure 6: The tree of the first row rooted tree by c_{12} .

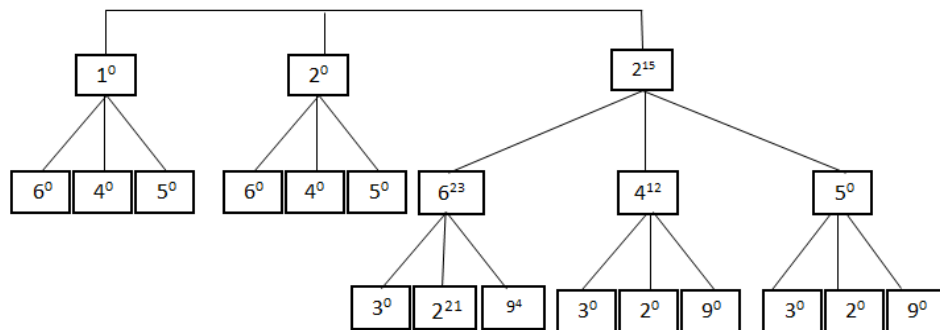


Figure 7: The tree of the first row rooted tree by c_{13} .

Source	Destination			Supply
	D_1	D_2	D_3	
s_1	1	2	2	15
s_3	3	2	9	25
s_2	6	4	5	35
Demand	23	33	19	75

Figure 8: The optimal solution based on OS.

We repeat the application of the algorithm on each element of the cost matrix, and using the MATLAB 2013 program, we obtain the optimal solution as in Figure 11.

Then, $Z = (15*1) + (8*3) + (17*2) + (16*4) + (19*5) = 232$ units, which is the optimal solution.

Source	Destination				supply
	D_1	D_2	D_3	D_4	
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	115

Figure 9: The transportation problems.

Using the proposed algorithm, programmed in MATLAB 2013, we obtained a Rooted Tree, which gives us the optimal solution. We moved the fourth column to become the second while maintaining the allocation for each column and row. As a result, we obtained the following Figure 10.

Source	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	1	4	3	2	30
S_2	7	5	2	6	50
S_3	4	7	2	3	35
Demand	15	45	25	30	115

Figure 10: The lowest possible cost.

We repeat the application of the algorithm to each element of the cost matrix to obtain the lowest possible cost by using MATLAB 2013, we get the lowest possible cost from the given Figure 12.

Then, $Z = (15 * 1) + (15 * 2) + (45 * 5) + (5 * 2) + (20 * 2) + (15 * 3) = 365$ units.

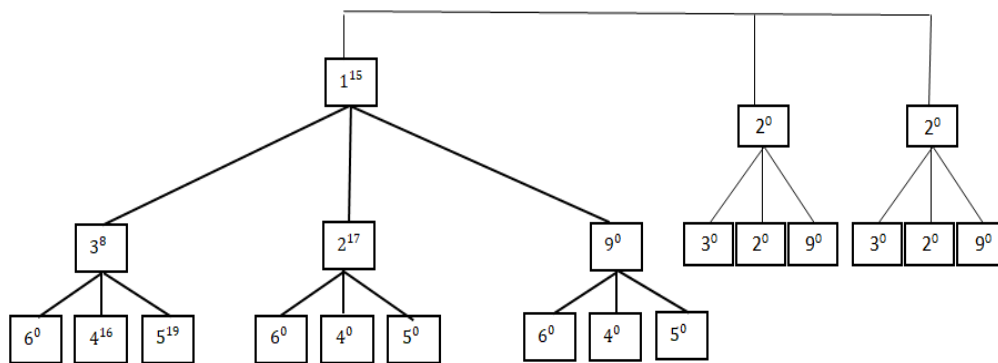
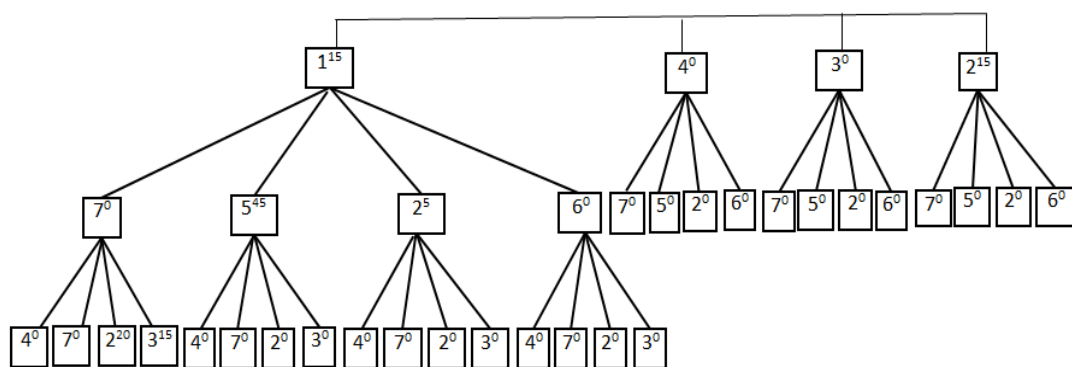

 Figure 11: The tree of the first row rooted tree by c_{11} .


Figure 12: The tree of the first row rooted tree.

5 COMPARISON

The outcomes of the instance solving conducted in the current study were compared to prove the workability of the new technique. The comparison has been conducted between the classic methods of solution (NWCM, LCM, VAM and method Al-Saeedi1). A superior IBFS is produced by the proposed technique, rather than the traditional algorithms. This algorithm has close or ideal positions to optimize the IBFS that is obtained by the current suggested algorithm. This algorithm is identified as superior to those acquired by employing Vogel's Approximation approach. The following Figure 13 shows the difference between methods to solve TP in the cost function.

Name of method	NWCM	LCM	VAM	Saeedi1	The proposed algorithm	OS
Example1	354	240	247	247	232	232
Example2	480	405	375	405	365	365

Figure 13: The difference between the methods for solving TP in the cost function.

6 CONCLUSIONS

The results from using the tree approach in this work are found to be better than the results obtained with the three traditional methods and the Saeedi method 1. Given what was shown earlier, it is clear that the new algorithm provides satisfactory and preferred solutions. As a result, the TP was solved using an objective function that was designed to be minimized. The understanding and use of the new algorithm are both considered to be simple in many cases. It saved me both time and energy in discovering the optimal solution (OS). On the basis of the best answer, we achieve a wise selection. Consequently, our proposed approach gives the method's algorithm a potential improvement by enhancing computational efficiency, reducing overall complexity, and facilitating easier implementation. These advantages make it particularly suitable for practical applications in industries where rapid and

cost-effective solutions are essential for maintaining competitiveness.

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