# Transient Phenomena in Information Technology for Branching Processes with an Infinite Set of Types

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Abstract: Branching processes as a mathematical concept has applications in various fields, including information technology. In information technology, branching processes can be used to model and analyze various scenarios, such as the propagation of data or information in a network, the growth of computer viruses, the spread of software bugs, and more. Branching processes are particularly useful for understanding the dynamics of systems where events can lead to multiple new events in a probabilistic manner. Overall, branching processes provide a valuable mathematical framework for modeling and analyzing various aspects of information technology, helping to make informed decisions and optimize IT systems and networks. We have studied transient phenomena for branching processes with an infinite number of types close to critical. The analytical apparatus for this study is Markov renewal theorems. Branched processes were used to evaluate the performance of IT systems and predict their behavior under different conditions. This is important for capacity planning and resource allocation.

## 1 INTRODUCTION

Branching processes are one of the most interesting sections of probability theory. The theory of branching processes has now become a much branched field of probability theory and a powerful research tool in various areas of mathematics, such as the theory of algorithms, queuing theory, random mapping theory, the theory of leakage, as well as in many branches of other sciences, which include, in particular, physics, chemistry, biology and information technology.

The theory of branching random processes is an important method in the theory of Markov queuing models. Similar processes arise, in particular, in the description of queuing systems (for example, in multiprocessor or distributed computer processing of data). This apparatus is especially important in the theory of reliability, where it is used to describe failures in complex equipment (failure of one element gives rise to failures of other elements).

Here are some ways branching processes can be applied in IT:

1) Network Propagation: In the context of computer networks and communication systems, branching processes can model how data packets or information propagate through the network. This is useful for understanding network congestion, data transfer rates, and the spread of information in social networks [1-8].
2) Virus and Malware Propagation: Branching processes can be used to model the spread of computer viruses, malware, and other malicious software within a network. This can help in analyzing the potential impact of a malware outbreak and developing strategies to mitigate it [9-10].
3) Software Development and Bugs: In software development, branching processes can be applied to model the occurrence and propagation of software bugs and issues. This can aid in understanding the factors that contribute to software quality and reliability [11-13].
4) Fault Tolerance and Redundancy: In designing fault-tolerant systems, branching processes can be used to model the reliability of components within a system. This helps in making informed

decisions about redundancy and backup strategies [14-18].

5) Queuing Systems: In IT systems, queuing models often involve branching processes to analyze the flow of requests or tasks through a system, such as in call centers, web servers, and database management [19-21].
6) Social Media Engagement: In the context of social media platforms and online communities, branching processes can be used to model the spread of content (e.g., viral videos, news articles) and user engagement patterns, helping to optimize content distribution strategies.
7) Performance Analysis: Branching processes can be used to assess the performance of IT systems and predict their behavior under different conditions. This is valuable for capacity planning and resource allocation [22-27].

## 2 TRANSIENT PHENOMENA OF MARKOV RENEWAL THEORY

The basic statements of the classical renewal theory can be extended to the so-called Markov renewal equation

$$f(x,t) = g(x,t) + \\ + \int_E \int_0^t G(x, dy \times du)\, f(y, t-u), t \geq 0, x \in E.$$

where $E$ is a given phase space, $G(x, dy \times du)$ is so-called semi-Markov kernel, $g(x,t)$ is a given function of $x \in E$, and $t \geq 0$, and $f(x,t)$ is the function to be found. Its solution is the convolution

$$f(x,t) = U * g(x,t) = \\ = \int_E \int_0^t U(x, dy \times du)\, g(y, t-u), t \geq 0, x \in E.$$

where $U(x, dy \times du)$ is the potential of the semi-homogeneous kernel $G(x, dy \times du)$.

Generally, the renewal theory has wide range of applications in mathematical practice. Markov renewal theorems are an analytical tool for studying the limiting behavior of Markov and related processes, including semi-Markov and regenerative processes.

For example we can consider Crump-Mode-Jagers branching process [31] with an arbitrary set of types (say $E$). Denote by $M_t(x, A)$ the conditional mean number of particles at moment $t \geq 0$ whose types belong to set $A \subset E$ under the condition that there was one new-born particle of type $x \in E$ at the initial moment $t_0 = 0$.

It is well-known that $M_t(x, A)$ as function of $x \in E$ and $t \geq 0$ satisfies the

$$M_t(x, A) = L_t(x, A) + \\ + \int_E \int_0^t K(x, dy \times du)\, M_{t-u}(y, A),$$

where, roughly speaking, $K(x, dy \times du)$ is equal to the conditional mean number (under the same condition) of particles of type $y$ produced by the initial particle during an infinitely small interval $[u, u + du)$, and $L_t(x, A)$ is equal to the conditional probability ( under the above condition) that the type of the still existing initial particle belongs to the set $A$.

W. Feller introduced the very important notion of direct Riemann integrability.

Namely, a Borel function $g(t), t \geq 0$, is called directly Riemann-integrable if

$$\sum_{k=0}^{\infty} \sup_{k \leq t \leq k+1} |g(t)| < \infty, \quad (1)$$

and

$$\delta \sum_{k=0}^{\infty} \left\{ \sup_{k\delta \leq t \leq k\delta+\delta} g(t) - \inf_{k\delta \leq t \leq k\delta+\delta} g(t) \right\} \underset{\delta \to 0}{\longrightarrow} 0. \quad (2)$$

Under conditions (1) and (2) the function $g(t)$ is absolutely integrable on $[0, \infty)$ and

$$\int_0^\infty g(t)dt = \lim_{\delta \to 0} \left[ \delta \sum_{k=1}^{\infty} g(t_k) \right],$$

where $t_k \in [k\delta, k\delta + \delta]$, in contrast to the usual definition of the improper Riemann integral as a limit of integrals over finite intervals.

That is why such a function $g(t)$ is called directly Riemann-integrable.

In the theory of branching processes in the case of a finite phase space, the Markov renewal equations are often considered

$$f_i(t) = g_i(t) + \sum_{j=1}^{d} \int_0^t f_j(t-u)\, dM_{ij}(u), \quad (3)$$

where the functions $M_{ij}(u)$ do not decrease, have bounded variation, and $M_{ij}(0) = 0$.

The asymptotic properties of the solution to this equation are essentially determined by the maximum eigenvalue $\lambda$ (the Perron root) of matrix

$$M(\infty) = \left\| M_{ij}(\infty) \right\|_{i,j=1}^{d}.$$

Equations (3) is called supercritical if $\lambda > 1$, critical, if $\lambda = 1$, and subcritical, if $\lambda < 1$. Critical equations with indecomposable matrix $M(\infty)$ simply reduce to the equations of Markov renewal. If the Perron root of an indecomposable matrix $M(\infty)$ is equal to one, then it has left and right invariant vectors $v = (v_1, v_2, \dots, v_d)$ and $u = (u_1, u_2, \dots, u_d)$ respectively with positive coordinates, i.e.

$$\sum_j M_{ij}(\infty)u_j = u_i, \quad \sum_i v_i M_{ij}(\infty) = v_j.$$

In the case when the criticality parameter $\lambda$ of (3) is close to one, but possibly not equal to it, then the problem of transient phenomena arises. One of the exact formulations of this problem is as follows. Let $M^{(n)}(t) = \left\| M_{ij}^{(n)}(t) \right\|_{i,j=1}^{d}, n = 1,2, \dots$, is a sequence $d \times d$ matrices, whose elements are non-decreasing functions of bounded variation with $M_{ij}^{(n)}(0) = 0$. Let also for all $i,j = 1,2, \dots, d$ the sequence $M_{ij}^{(n)}(t), n = 1,2, \dots$, converges to $M_{ij}(t)$ at the points of continuity of the limit function, and the matrix $M(\infty) = \left\| M_{ij}(\infty) \right\|_{i,j=1}^{d}$ is indecomposable and its Perron root is equal to one. This assumption allows, without loss of generality, to assume that the matrices $M^{(n)}(\infty)$ are also not decomposable, and that their Perron roots $\lambda_n$ converge as $n \to \infty$ to one, and the right and left eigenvectors $u^{(n)}$ and $v^{(n)}$ converge respectively to the right-left invariant vectors $u = (u_1, u_2, \dots, u_d)$ and $v = (v_1, v_2, \dots, v_d)$ of the matrix $M(\infty)$.

Let $f_i^{(n)}(t)$ at each $n = 1,2, \dots$ be a solution of the

$$f_i^{(n)}(t) = g_i^{(n)}(t) + \sum_{j=1}^{d} \int_0^t f_j^{(n)}(t-u)\, dM_{ij}^{(n)}(u),$$

functions sequence $g_i^{(n)}(t),\ n = 1,2, \dots$, is uniformly directly Riemann integrable on $[0, \infty)$. Then if

$$\sup_{n \ge 1} \int_t^\infty s dM_{ij}^{(n)}(s) \underset{t \to \infty}{\longrightarrow} 0,$$

and the matrix $M(t)$ is nonlattice, then

$$f_i^{(n)}(t) - \frac{u_i}{m} e^{c/m} \sum_j v_j \int_0^\infty g_j^{(n)}(u) du \to 0,$$

as $t \to \infty, n \to \infty, t(\lambda_n - 1) \to c$, where

$$m = \sum_{ij} v_i \int_0^\infty t dM_{ij}(t) u_j, \quad \sum_j v_j u_j = 1.$$

In the infinite-dimensional case, let $f_\varepsilon(x,t)$ at every $\varepsilon > 0$ be a solution of the Markov renewal equation

$$f_\varepsilon(x,t) = g_\varepsilon(x,t) +$$

$$+ \int_E \int_0^t G_\varepsilon(x, dy \times du)\, f_\varepsilon(y, t-u), t \ge 0, x \in E.$$

Transient phenomena for the solution of the Markov renewal equation

$$f_\varepsilon(x,t) = U_\varepsilon * g_\varepsilon(x,t) =$$

$$= \int_E \int_0^t U_\varepsilon(x, dy \times du)\, g_\varepsilon(y, t-u), t \ge 0, x \in E,$$

as $t \to \infty, \varepsilon \to 0, t(1-\lambda_\varepsilon)/m \to c$, where $\lambda_\varepsilon$ is the Perron root of the basis $G_\varepsilon(x,dy)$ of the kernel $G_\varepsilon(x, dy \times dt)$,

$$m = \int_E \int_E \int_0^\infty l(dx) G(x, dy \times dt) h(y) t,$$

$$\int_E l(dx) h(x) = 1,$$

$h$ and $l$ are eigenfunction and eigenmeasure respectively of the basis of the limit semi-homogeneous kernel $G(x, dy \times dt)$, were investigate [28].

We denote $\gamma_\varepsilon = (1-\lambda_\varepsilon)/m$. In [28] it was proved, if

1) the basis $G(x,dy)$ of the kernel $G(x, dy \times dt)$, that is $G(x,dy) = G\big(x, dy \times [0,\infty)\big)$, is conservative [32], and its Perron root equals 1,
2) there exists a Borel function $g(t)$ such that

$$\int_0^\infty \left| g(t) - \int_E l(dx) g_\varepsilon(x,t) \right| dt \underset{\varepsilon \to 0}{\longrightarrow} 0,$$

then

$$\lim_{\substack{\varepsilon \to 0 \\ t \to \infty \\ \gamma_\varepsilon t \to c}} U_\varepsilon * g_\varepsilon(x,t) = e^{-c} \frac{h(x)}{m} \int_0^\infty g(s) ds,$$

uniformly in $x \in E$.

The another polar case is the degeneracy of the basis of the limit kernel. The asymptotics of the solution of the Markov renewal equation when the basis $G_\varepsilon(x,dy) = G_\varepsilon(x,dy \times [0,\infty))$ of the kernel $G_\varepsilon(x,dy \times dt)$ close to the singular kernel $I(x,dy)$ on a given measurable phase space $(E,\mathfrak{B})$ was studied in [29].The main result of that study was formulated in the form of a theorems.

The formal definition of branching processes with an arbitrary number of types of particles, the transformations of which may depend on their age, is rather cumbersome. By contrast, a descriptive description is simple and short. Any such branching process is associated with the following evolving population, consisting of particles of several types: each of the particles existing at a given time, regardless of its origin and the presence of other particles, after the expiration of its existence, turns into a certain (possibly empty) set of newborn particles. The progeny of a particle depends only on its type and the age at which the transformation took place.

Critical processes have the most interesting asymptotic properties. However, the question of the criticality of the actually observed branching process is not simple and it is not always possible to give an unambiguous answer to it. From this point of view, it is very important to study the asymptotic of branching processes, when over time the criticality parameter (in this case, the Perron root) tends to one. The resulting phenomena are called transient.

This article studies transient phenomena for branching processes with an infinite number of types close to critical. The analytical apparatus for this study is Markov renewal theorems. The asymptotic properties of the solution of the Markov renewal equation were studied in [32].

# 3 A BRANCHING PROCESS WITH AN INFINITE SET OF TYPES

First, we describe a model for the evolution of a population with an infinite number of types.

Let be $E$ - abstract set, which we will call the set of types.

Suppose that on the set $E$ is distinguished a $\sigma$-algebra of its subsets $\mathfrak{B}$ which contains all one-point sets and, moreover, is generated by a countable number of its elements.

A population is considered, which consists of a certain number of particles, for each of which a certain type is assigned, that is, an element of the set $E$. The law of population evolution is as follows. A newborn particle of type $x$, regardless of the presence of other particles and the previous history of the development of the population, lives a random time $\tau(x) > 0$, at the end of which it turns into some (possibly empty) set of newborn particles of different types.

Let us denote $(\Omega, \mathrm{M}, \mathbb{P})$ - the basic probability space; $\zeta_t(x)$ - is the type of particle at the end of $t$ units of its lifetime, $\zeta_0(x) = x, 0 \le t < \tau(x)$; $\eta(x,S)$ - is the number of immediate descendants with types from the set $S \in \mathfrak{B}$ of one particle that had type $x$, $\xi_t(x,S,v)$ - is the number of particles formed during time $t$ from one newborn particle of the type $x$ whose age is at least $v$ and whose types belong to the set $S \in \mathfrak{B}$.

Suppose that all introduced characteristics depend measurably on the set of variables $x \in E,\ t \ge 0, \omega \in \Omega$.

Let us fix the number $v > 0$, bounded non-negative $\mathfrak{B}$ - measurable function $\varphi$ and put

$$f(x,t) = A(x,t) = \mathbb{E} \int_E \xi_t(x,dy,v)\varphi(y),$$

$$g(x,t) = 0 \ at\ 0 \le t < v,$$

$$g(x,t) = \mathbb{E}\left[\varphi(\zeta_t(x)) I_{\{t<\tau(x)\}}\right]\ at\ \ t \ge v,$$

$$G(x,dy \times du) = \mathbb{E}\left[\eta(x,dy) I_{\{\tau(x)\in du\}}\right].$$

Here and in what follows, the symbol $\mathbb{P}$ is denoted the main probability measure and the symbol $\mathbb{E}$ denoted corresponding mathematical expectation.

By slightly modifying the thinking from [30], we can show that the function $A(x,t)$ satisfies the Markov renewal type equation

$$A(x,t) = g(x,t) + \int_E \int_0^t G(x,dy \times du)\, A(y,t-u).$$

According to the total probability formula, we have

$$A(x,t) = \mathbb{E}\left[\varphi(\zeta_t(x)) I_{\{t<\tau(x)\}}\right] +$$

$$+ \int_0^t \mathbb{E}\left[\int_E \xi_t(x,dz,v)\varphi(z) I_{\{\tau(x)\in du\}}\right].$$

Further at $u < t$ we have

$$\mathbb{E}\left[\int_E \xi_t(x,dz,v)\varphi(z) I_{\{\tau(x)\in du\}}\right] =$$

$$= \mathbb{E}\left[\int_E \xi_{\tau(x)+t-u}(x, dz, v)\varphi(z) I_{\{\tau(x)\in du\}}\right] =$$

$$= \int_E \mathbb{E}\left[\eta(x, dy) I_{\{\tau(x)\in du\}}\right] \mathbb{E}\int_E \xi_{t-u}(y, dz, v)\varphi(z)$$

$$= \int_E G(x, dy\times du) A(y, t-u).$$

That's why

$$A(x,t) = \int_E \int_0^t U(x, dy\times du)\, g(y, t-u), \quad (4)$$

where

$$U(x, dy\times du) = \sum_{k=1}^{\infty} G^{k*}(x, dy\times du)$$

is recovery kernel that matches the kernel

$$G(x, dy\times du) = \mathbb{E}\left[\eta(x, dy) I_{\{\tau(x)\in du\}}\right].$$

The representation (4) allows one to give (by the same methods as in [30] a complete description limiting behavior of the mean $A(x,t)$ as $t\to\infty$ under the assumption that the kernel $G(x, dy) = G(x, dy\times[0,\infty))$ is critical and conservative (in [30] such kernels called critical and recurrent).

# 4 TRANSIENT PHENOMENA FOR BRANCHING PROCESSES WITH AN INFINITE SET OF TYPES CLOSE TO CRITICAL

We will assume that all introduced random variables depend on the small parameter $\varepsilon > 0$. Accordingly, we denote

$$\zeta_t(x) = \zeta_t^{\varepsilon}(x),\ \ \eta(x,S) = \eta^{\varepsilon}(x,S),$$

$$\xi_t(x,S,v) = \xi_t^{\varepsilon}(x,S,v),$$

$$\tau(x) = \tau^{\varepsilon}(x), \qquad x\in E, \qquad t\geq 0.$$

We fix a number $v > 0$, a bounded non-negative $\mathfrak{B}$ - measurable function $\varphi$ and put

$$f_{\varepsilon}(x,t) = A_{\varepsilon}(x,t) = \mathbb{E}\int_E \xi_t^{\varepsilon}(x, dy, v)\varphi(y),$$

$$g_{\varepsilon}(x,t) = 0\ at\ 0\leq t < v,$$

$$g_{\varepsilon}(x,t) = \mathbb{E}\left[\varphi\left(\zeta_t^{\varepsilon}(x)\right) I_{\{t<\tau^{\varepsilon}(x)\}}\right]\ \ at\ \ t\geq v.$$

The function $A_{\varepsilon}(x,t)$ satisfies the Markov renewal type equation

$$A_{\varepsilon}(x,t) = g_{\varepsilon}(x,t) + \\ + \int_E \int_0^t G_{\varepsilon}(x, dy\times du)\, A_{\varepsilon}(y, t-u), \qquad (5)$$

where

$$G_{\varepsilon}(x, dy\times du) = \mathbb{E}\left[\eta^{\varepsilon}(x,dy) I_{\{\tau^{\varepsilon}(x)\in du\}}\right], \\ x\in E, \qquad t\geq 0.$$

Thus, it can be argued that the solution of the equation (5) has the form

$$A_{\varepsilon}(x,t) = \int_E \int_0^t U_{\varepsilon}(x, dy\times du)\, g_{\varepsilon}(y, t-u),$$

where $U_{\varepsilon}(x, dy\times du)$ - the potential of the kernel $G_{\varepsilon}(x, dy\times du)$.

We will be interested in the asymptotic behavior $A_{\varepsilon}(x,t)$ at large values $t\geq 0$ and small values $\varepsilon > 0$.

We impose a number of the following conditions. Let the random process $\zeta_t^{\varepsilon}(x)$ converges to the random process $\zeta_t^0(x)$ with life time $\tau^0(x)$ particles with type $x$ in that sense

$$\lim_{\varepsilon\to 0} \mathbb{P}\left\{\zeta_{t_1}^{\varepsilon}(x)\in A_1,\dots,\zeta_{t_n}^{\varepsilon}(x)\in A_n, t_1,t_2,\dots,t_n < \tau^{\varepsilon}(x)\right\} = \mathbb{P}\left\{\zeta_{t_1}^0(x)\in A_1,\dots,\zeta_{t_n}^0(x)\in A_n, t_1,t_2,\dots,t_n < \tau^0(x)\right\} \qquad (6)$$

at points of continuity $t_1,\dots,t_n$ limiting probability distribution, $n = 1,2,\dots$, $A_i\in\mathfrak{B}$, $i = 1,2,\dots,n$, uniformly on $x\in E$.

We denote

$$G_{\varepsilon}(x,A) = \mathbb{E}\left[\eta^{\varepsilon}(x,A)\right]$$

and suppose that

$$\sup_{x\in E}\sup_{A\in\mathfrak{B}} |G_{\varepsilon}(x,A) - G(x,A)| \underset{\varepsilon\to 0}{\longrightarrow} 0, \qquad (7)$$

where

$$G(x,dy) = \mathbb{E}\left[\eta^0(x,dy)\right].$$

We will assume that the kernel $G(x,dy)$ is conservative and its Perron root is equal to one (the kernel is critical). This guarantees the existence of a non-trivial $\sigma$- finite measure $l$ and a positive $\mathfrak{B}$ - measurable $l$ - almost everywhere finite function $h$ such that

$$\int_E l(dx) G(x,A) = l(A),\ \ A\in\mathfrak{B},$$

$$\int_E G(x,dy)h(y) = h(x), \quad x \in E.$$

Let be

$$0 < \inf_{x\in E} h(x) < \sup_{x\in E} h(x) < \infty. \qquad (8)$$

From (6), (7) the existence of a set of measures $l_\varepsilon$ on $(E, \mathfrak{B})$ and functions $h_\varepsilon(x)$ is follows, such that

$$\int_E l_\varepsilon(dx)G_\varepsilon(x,A) = \lambda_\varepsilon l_\varepsilon(A), \quad A \in \mathfrak{B},$$

$$\int_E l_\varepsilon(dx)h_\varepsilon(x) = 1,$$

$$\sup_{A\in\mathfrak{B}} |l_\varepsilon(A) - l(A)| \underset{\varepsilon\to 0}{\longrightarrow} 0, \; h_\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} h,$$

where $\lambda_\varepsilon$ - is the Perron root of the kernel $G_\varepsilon(x,dy)$, $\lambda_\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} 1$.

Suppose the condition

$$0 < m = \int_E \int_E \int_0^\infty l(dx)G(x,dy\times dt)h(y)t < \infty. \quad (9)$$

Applying the Markov renewal theorem [28], we obtain the following statement.

**Theorem.** Let in (6) - (9) the kernel $G(x,dy)$ is critical and conservative, the kernel $G(x,dy\times du) = \mathbb{E}\left[\eta^0(x,dy)I_{\{\tau^0(x)\in du\}}\right]$ is non-lattice and a random process $\varphi(\zeta_t^\varepsilon(x))$ at $t < \tau^\varepsilon(x)$, stochastically continuous uniformly on $\varepsilon > 0$, $x \in E$, that is

$$\sup_{\varepsilon>0}\sup_{x\in E} \mathbb{P}\{|\varphi(\zeta_t^\varepsilon(x)) - \varphi(\zeta_u^\varepsilon(x))| > \delta, t, u < \tau^\varepsilon(x)\} \underset{t-u\to 0}{\longrightarrow} 0$$

for all δ>0, then, if

$$\sup_{\varepsilon>0} \int_E \int_E \int_T^\infty l(dx)G_\varepsilon(x,dy\times dt)h(y)t \underset{T\to\infty}{\longrightarrow} 0.$$

$$\sup_{\varepsilon>0}\sup_{x\in E}\sup_{t\geq 0} A_\varepsilon(x,t) < \infty, \int_E l(dx)\,\mathbb{E}\,\tau^0(x) < \infty,$$

$$\lim_{\varepsilon\to 0} \int_E l(dx) \int_v^\infty \mathbb{E}\left[\varphi\left(\zeta_t^\varepsilon(x)\right)I_{\{t<\tau^\varepsilon(x)\}}\right]dt =$$

$$= \int_E l(dx) \int_v^\infty \mathbb{E}\left[\varphi\left(\zeta_t^0(x)\right)I_{\{t<\tau^0(x)\}}\right]dt = l_v(\varphi),$$

then

$$\lim_{\substack{\varepsilon\to 0\\ t\to\infty\\ \gamma_\varepsilon t\to c}} A_\varepsilon(x,t) = e^{-c}\frac{h(x)}{m} l_v(\varphi),$$

uniformly on $x \in E$.

## 5 CONCLUSIONS

In branching processes, random variables are typically used to represent the number of offspring generated by each event, and the process can be analyzed using probability theory and stochastic processes. This allows IT professionals and researchers to make probabilistic predictions and decisions regarding system behavior and performance.

Overall, branching processes provide a valuable mathematical framework for modeling and analyzing various aspects of information technology, helping to make informed decisions and optimize IT systems and networks.

We have studied transient phenomena for branching processes with an infinite number of types close to critical. The analytical apparatus for this study is Markov renewal theorems.

Branched processes were used to evaluate the performance of service-oriented information technology to solve problems of sustainable environmental management and uniform information platform for the national automated ecological information and analytical system.

This is important for capacity planning and resource allocation.

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